Reliable Two-Dimensional Graphing Methods for Mathematical Formulae with Two Free Variables

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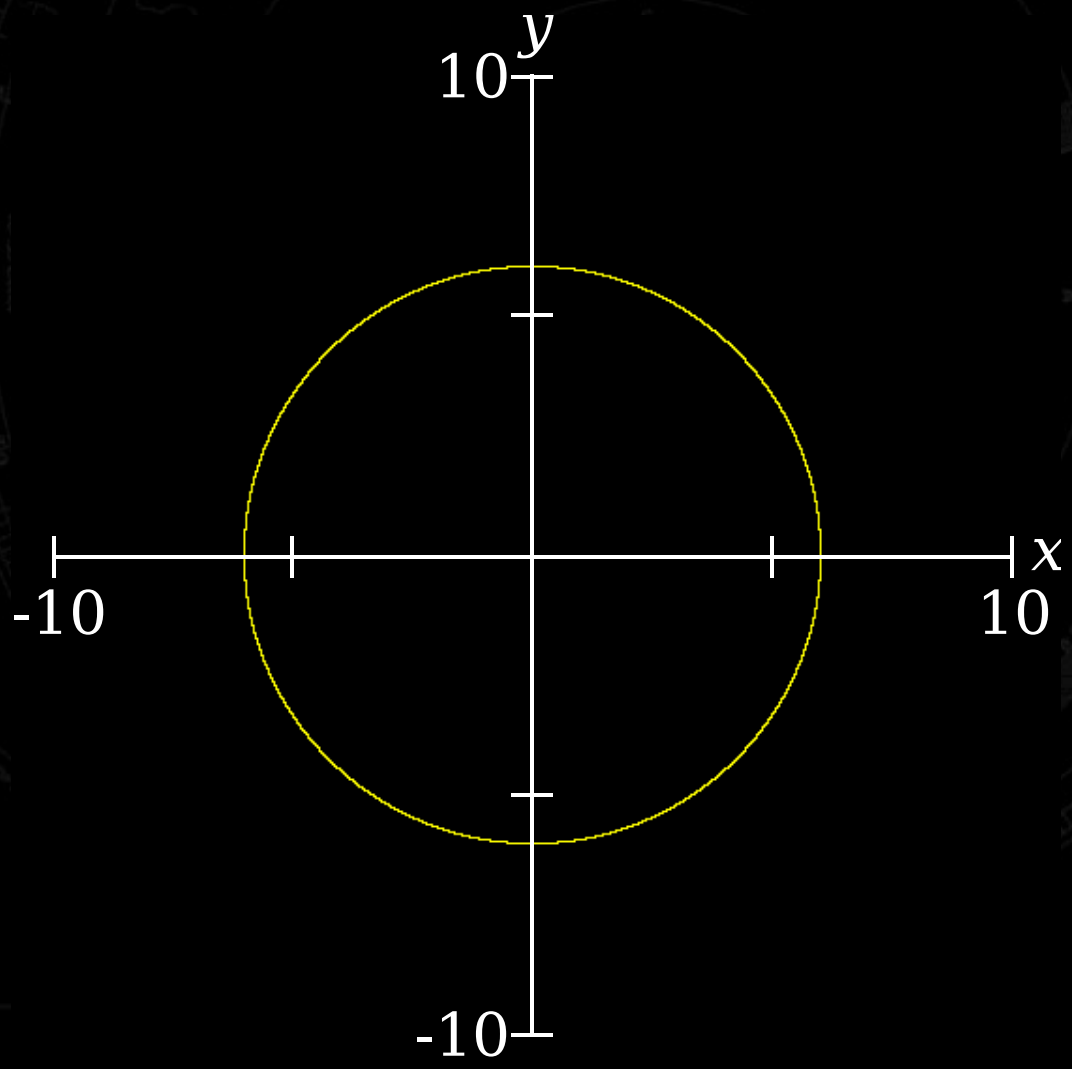
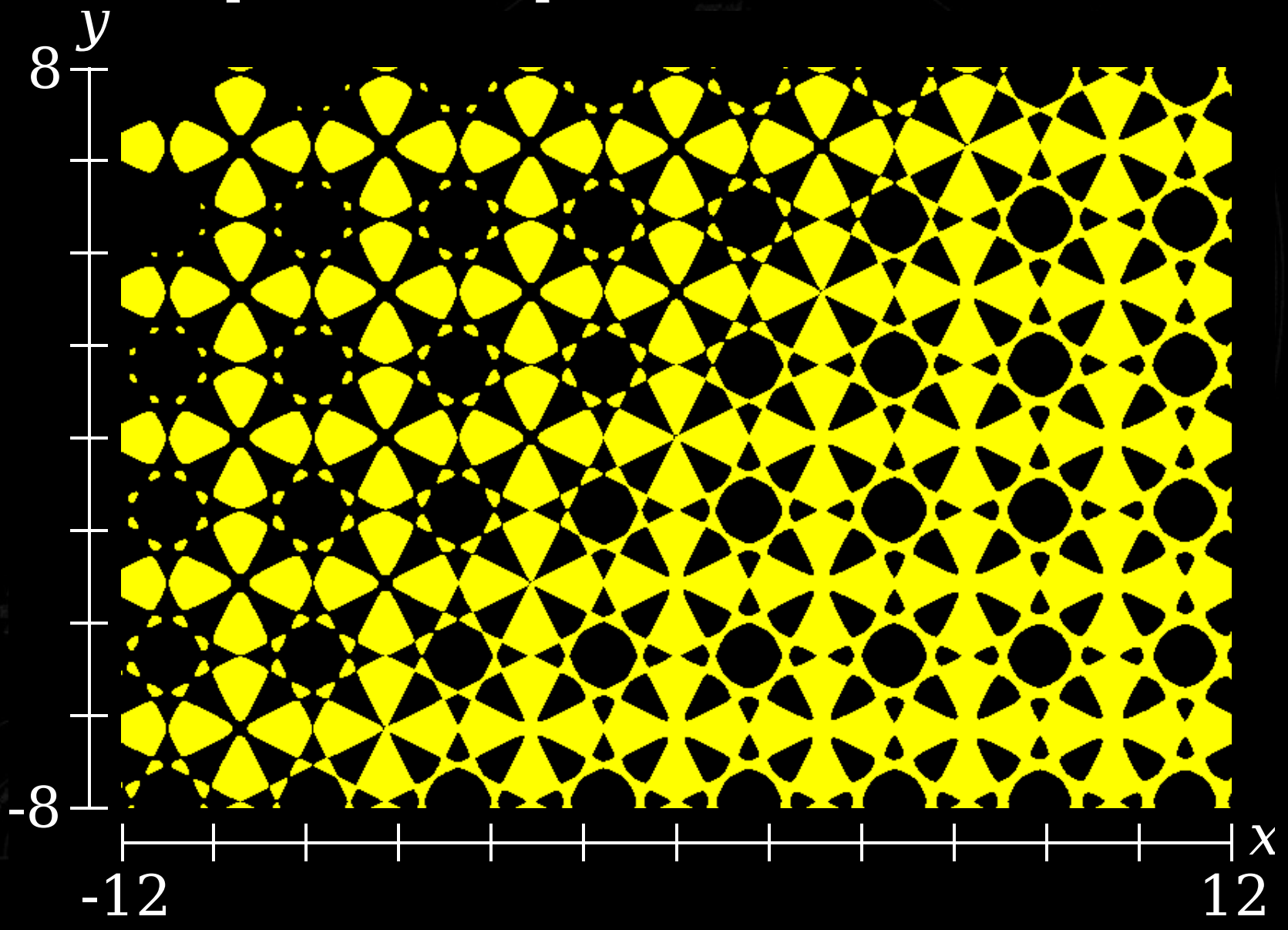
Dynamic Graphics Project
University of Toronto

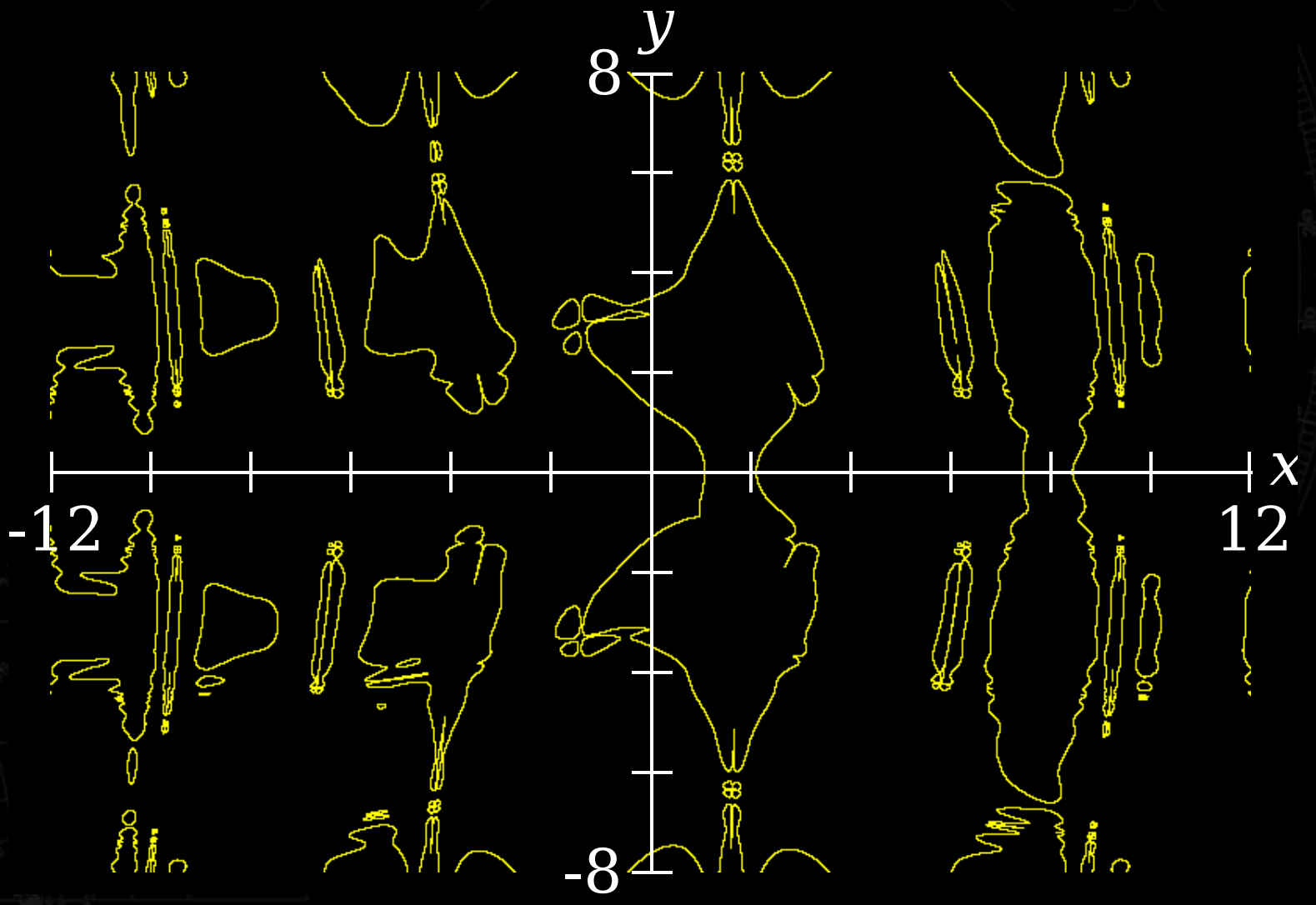
A Graph of a Circle



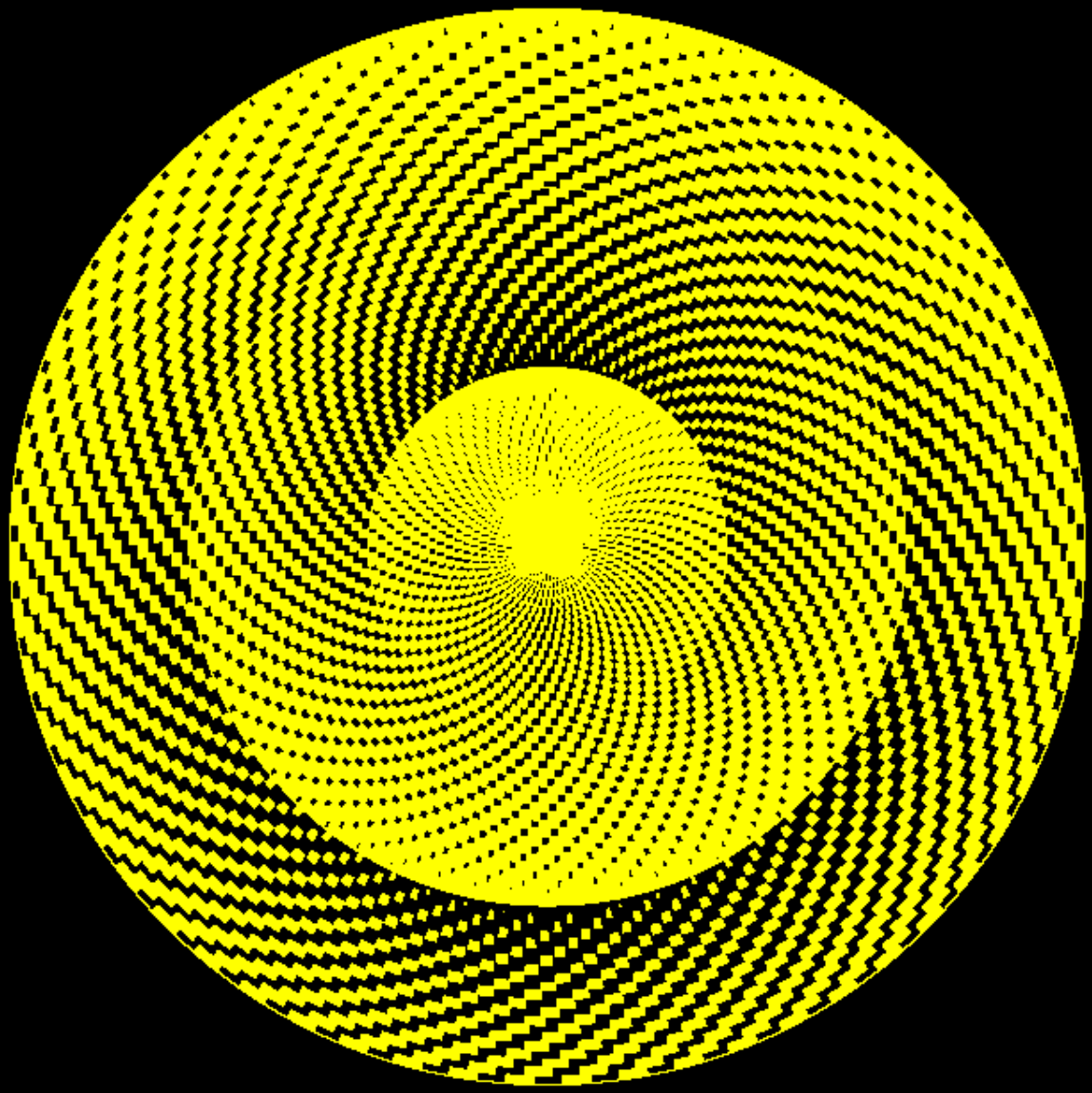
Example Graph



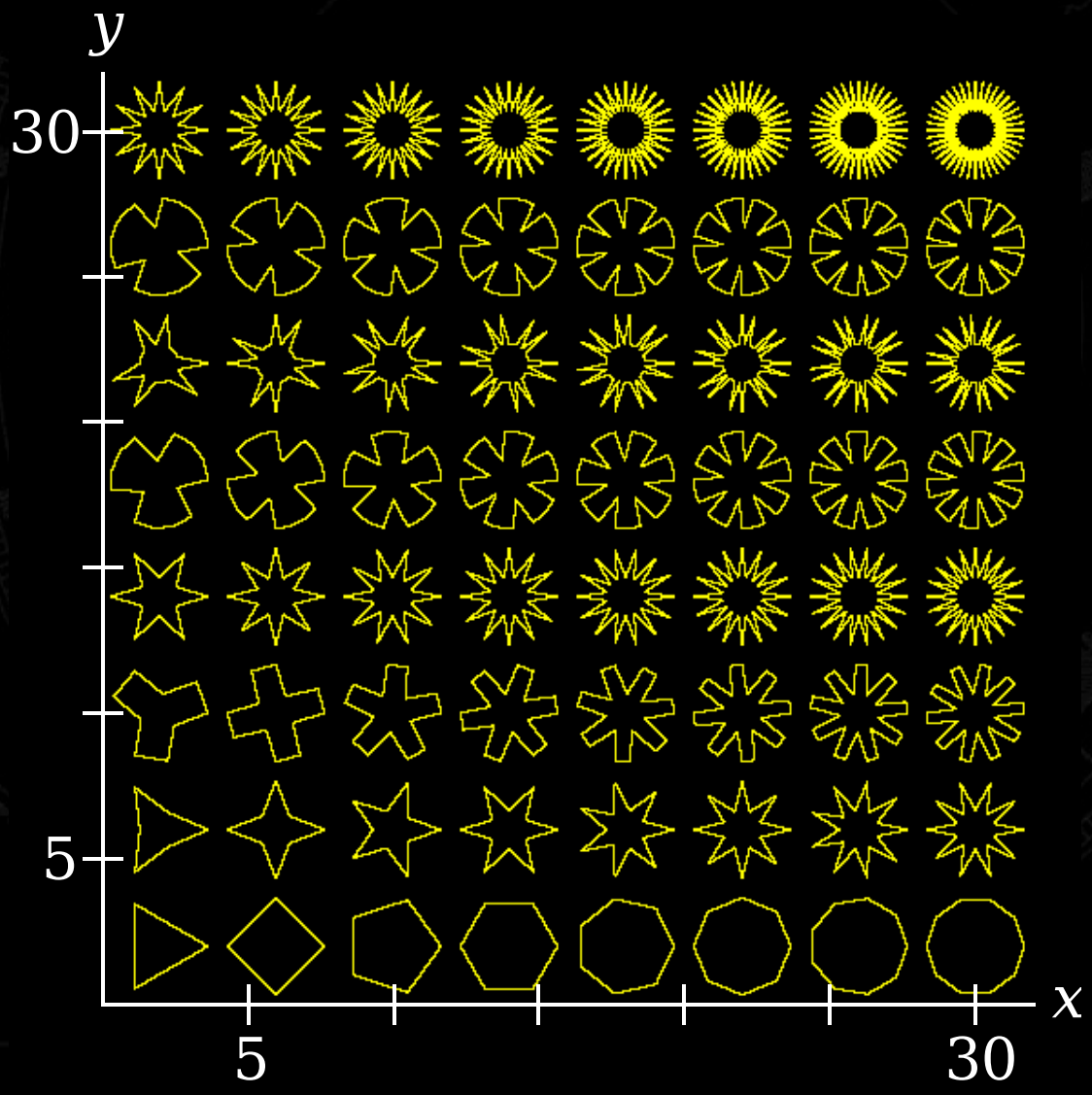
Example Graph



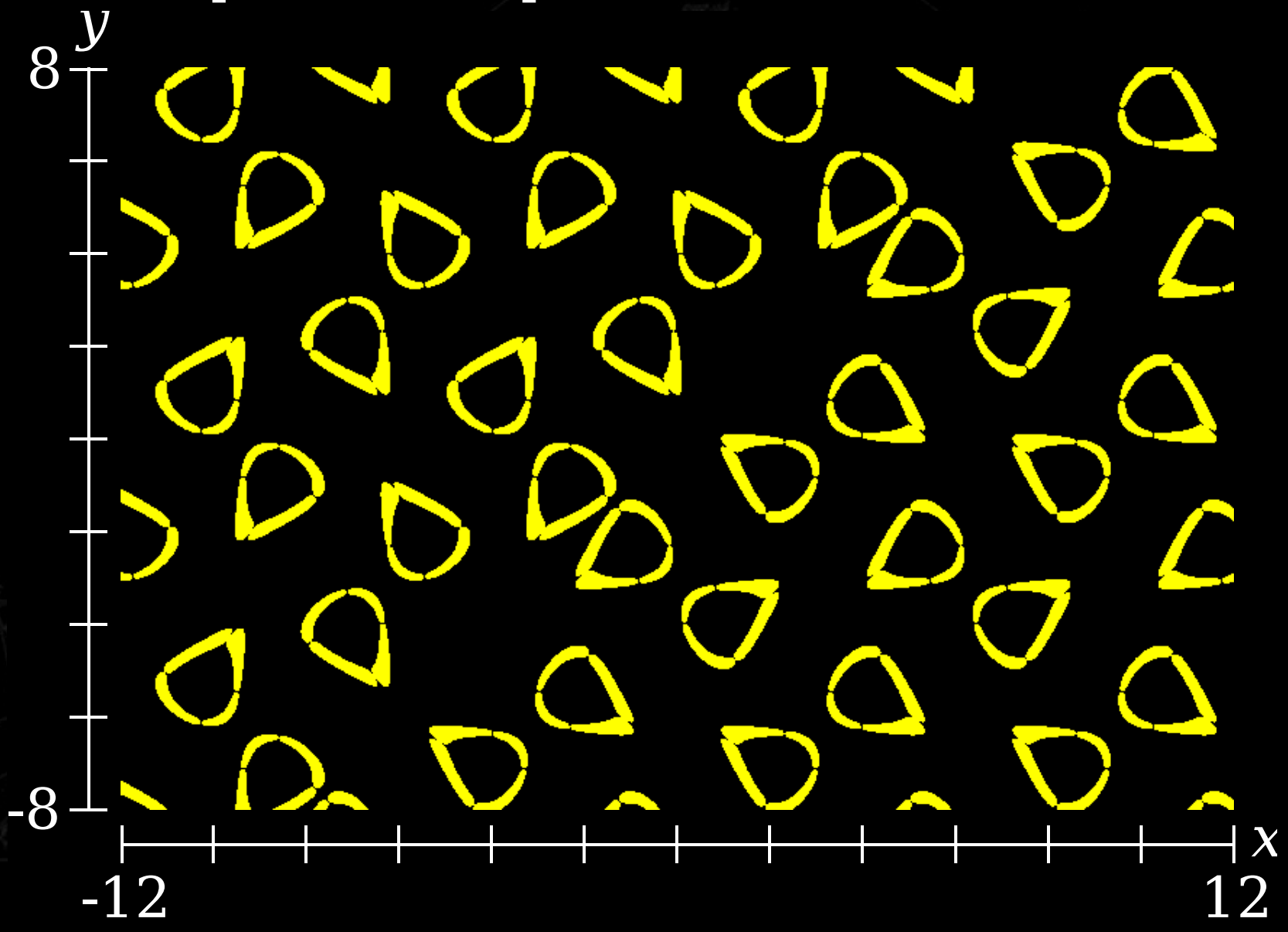
Example Graph



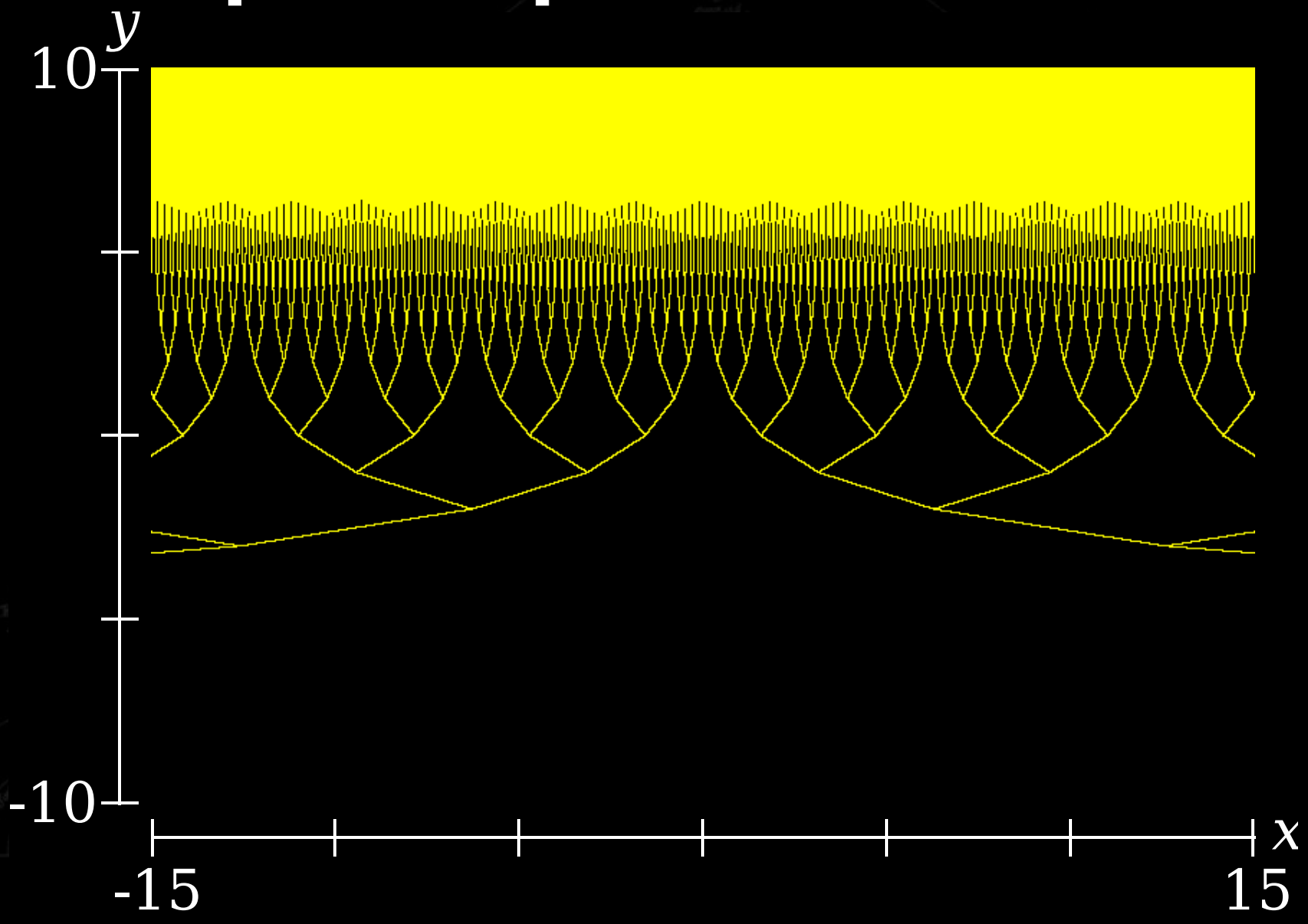
Example Graph



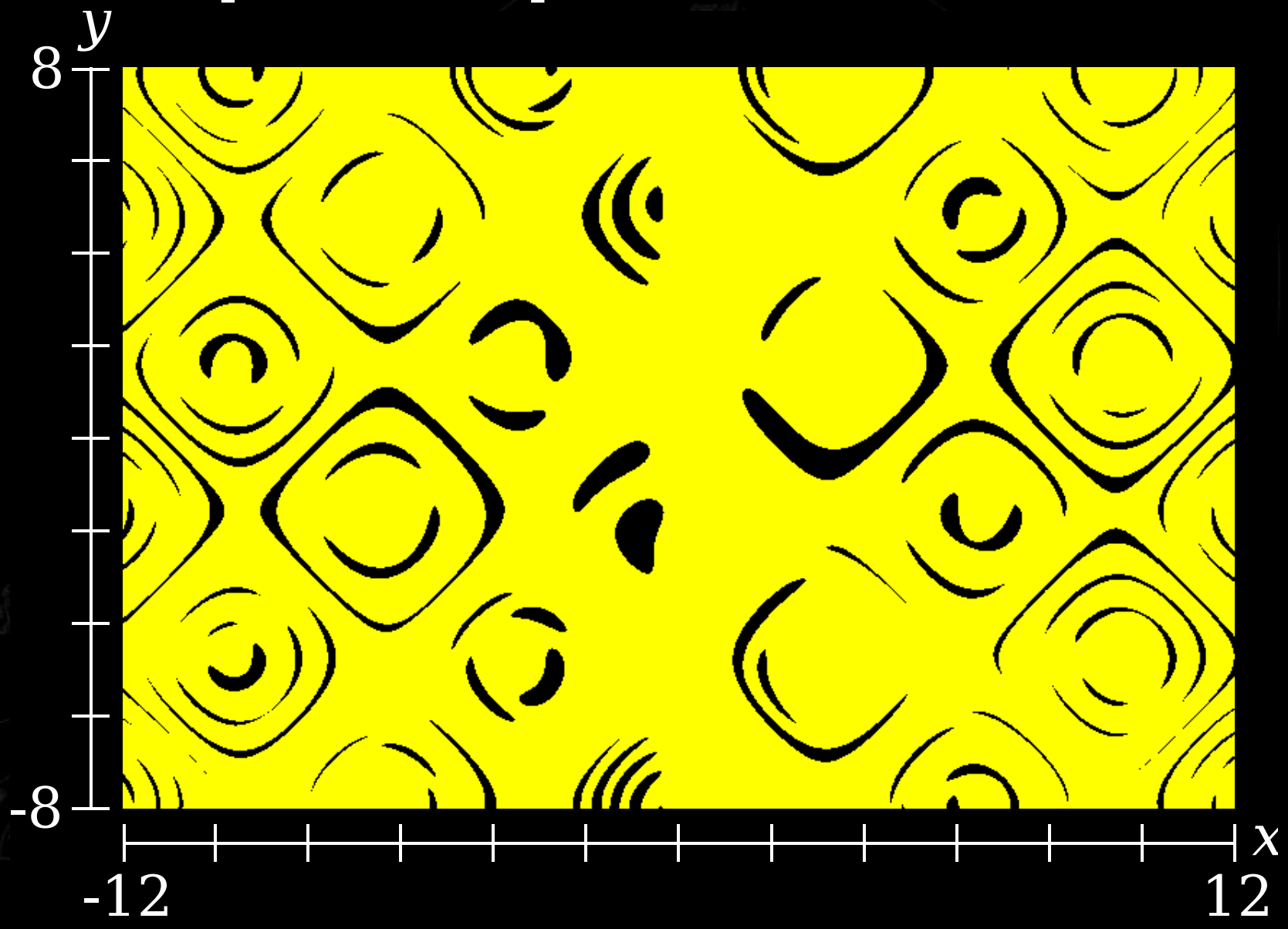
Example Graph



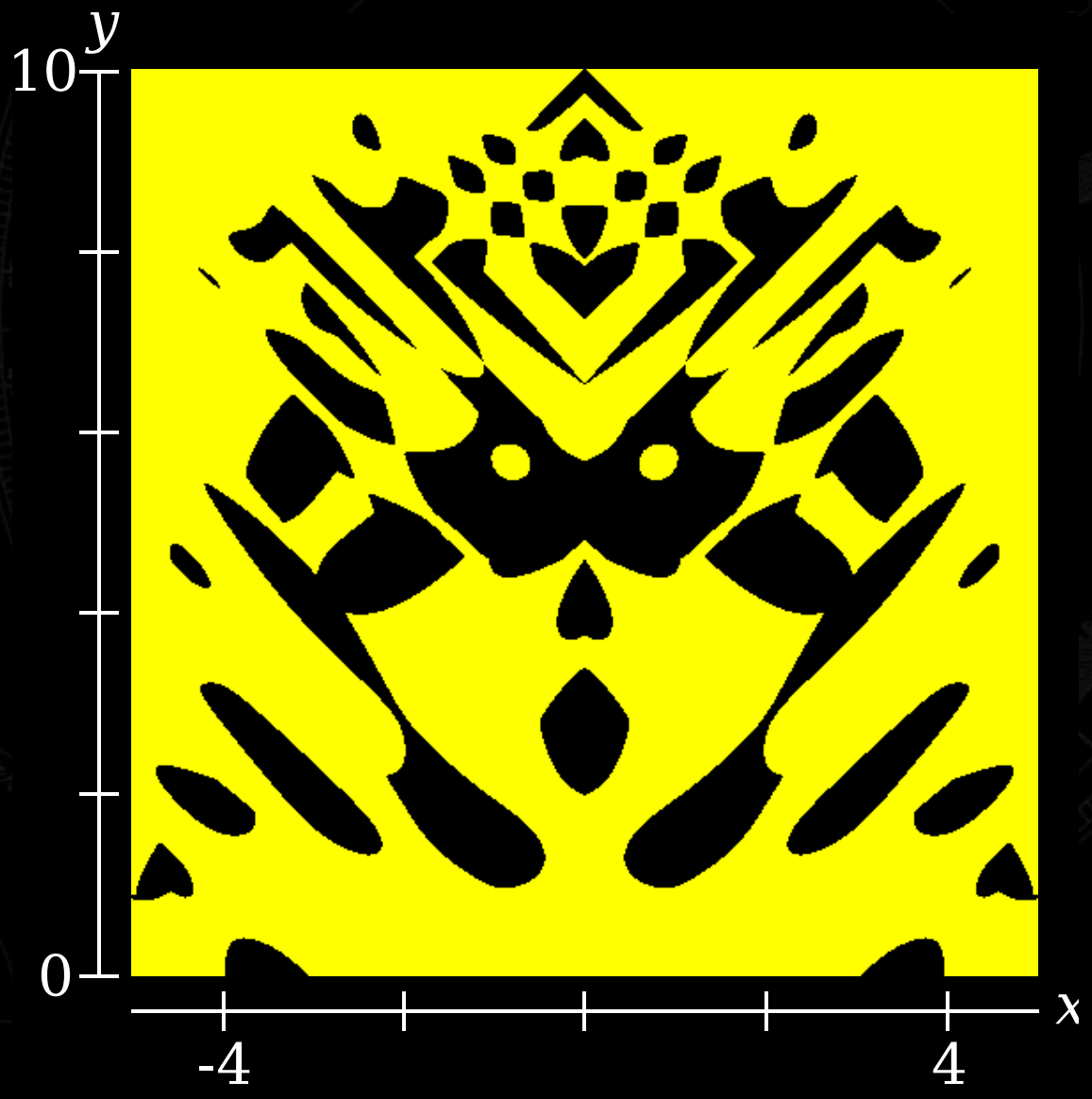
Example Graph



Example Graph



Example Graph

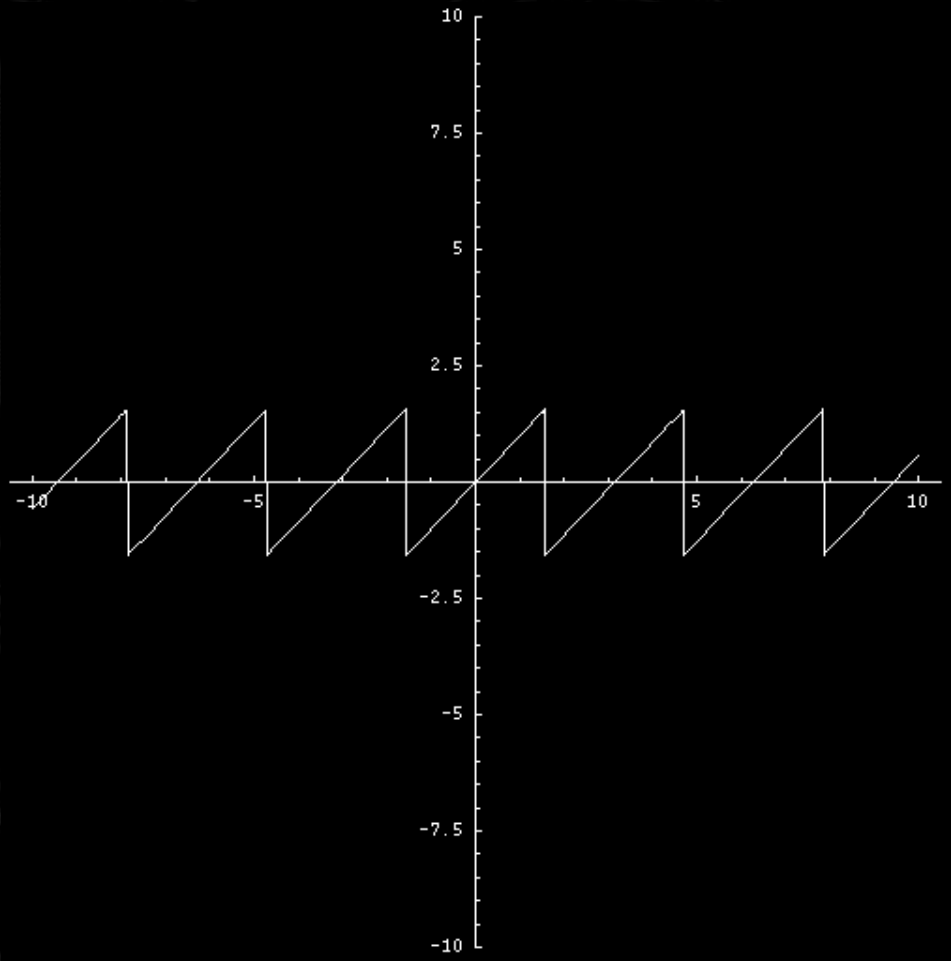


Isn’t this already solved?

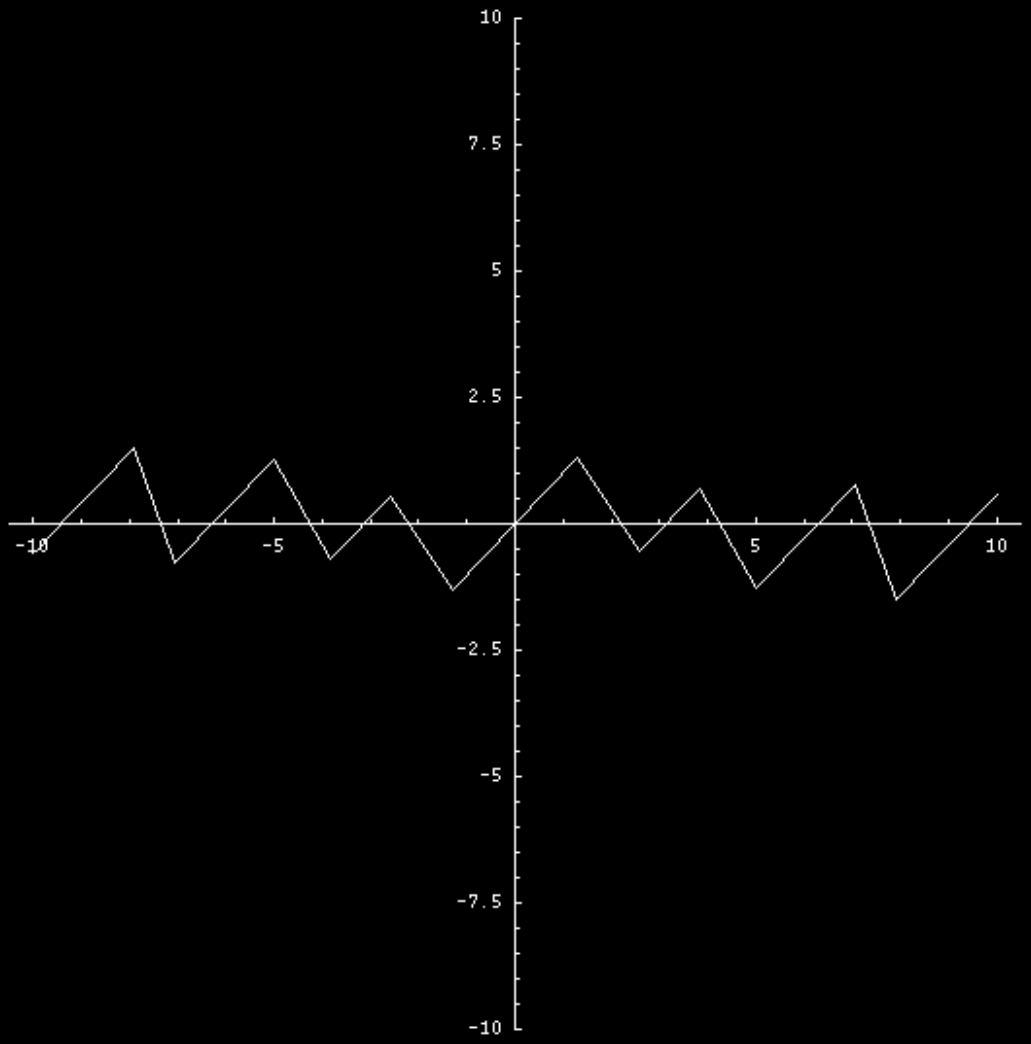
There are many utilities for doing this:

- **Computer Algebra Systems**
  - Mathematica, Maple, …
- **Graphing Calculators**
  - Hewlett-Packard, Texas Instruments, …
- **Graphing Software**
  - Curvus Pro, IAsolve, GraphingCalculator, …

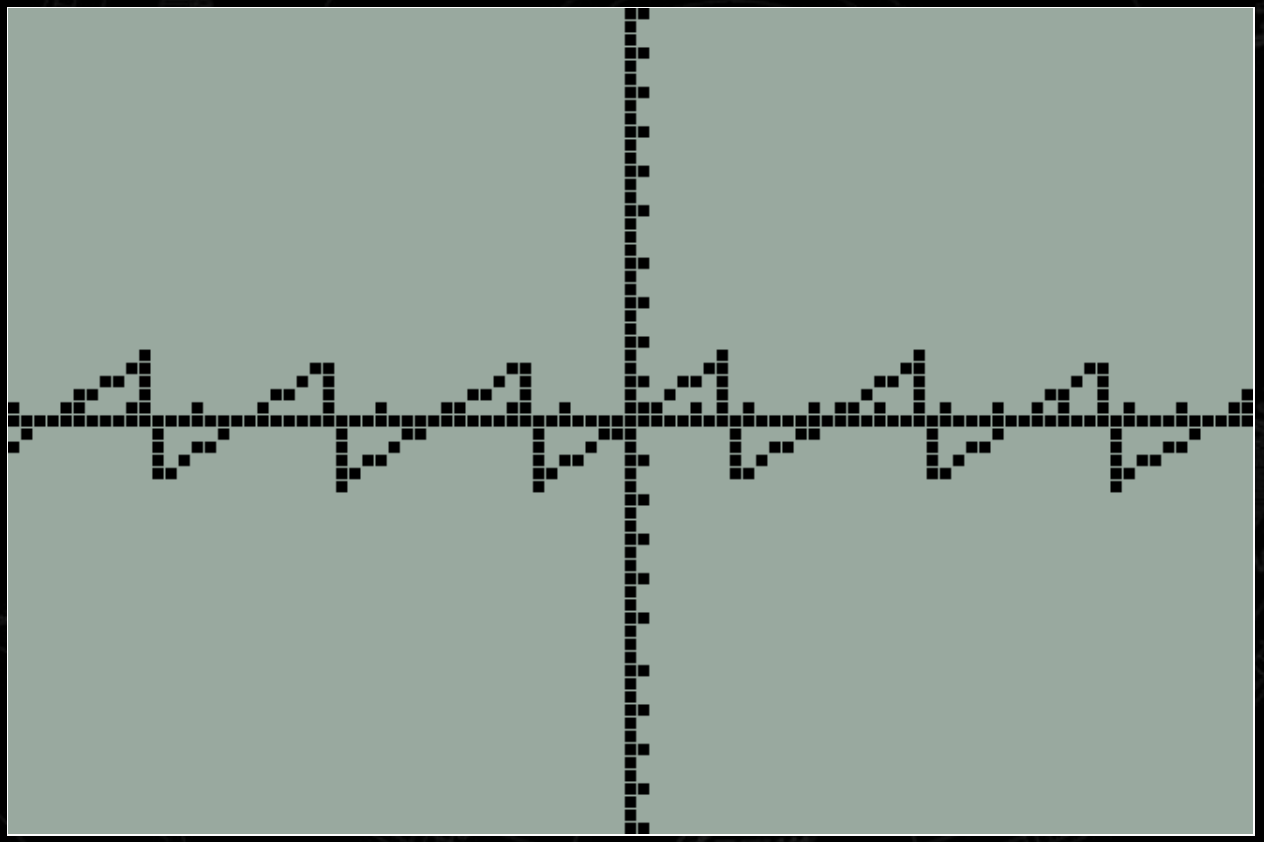
Mathematica 4 Plot Output



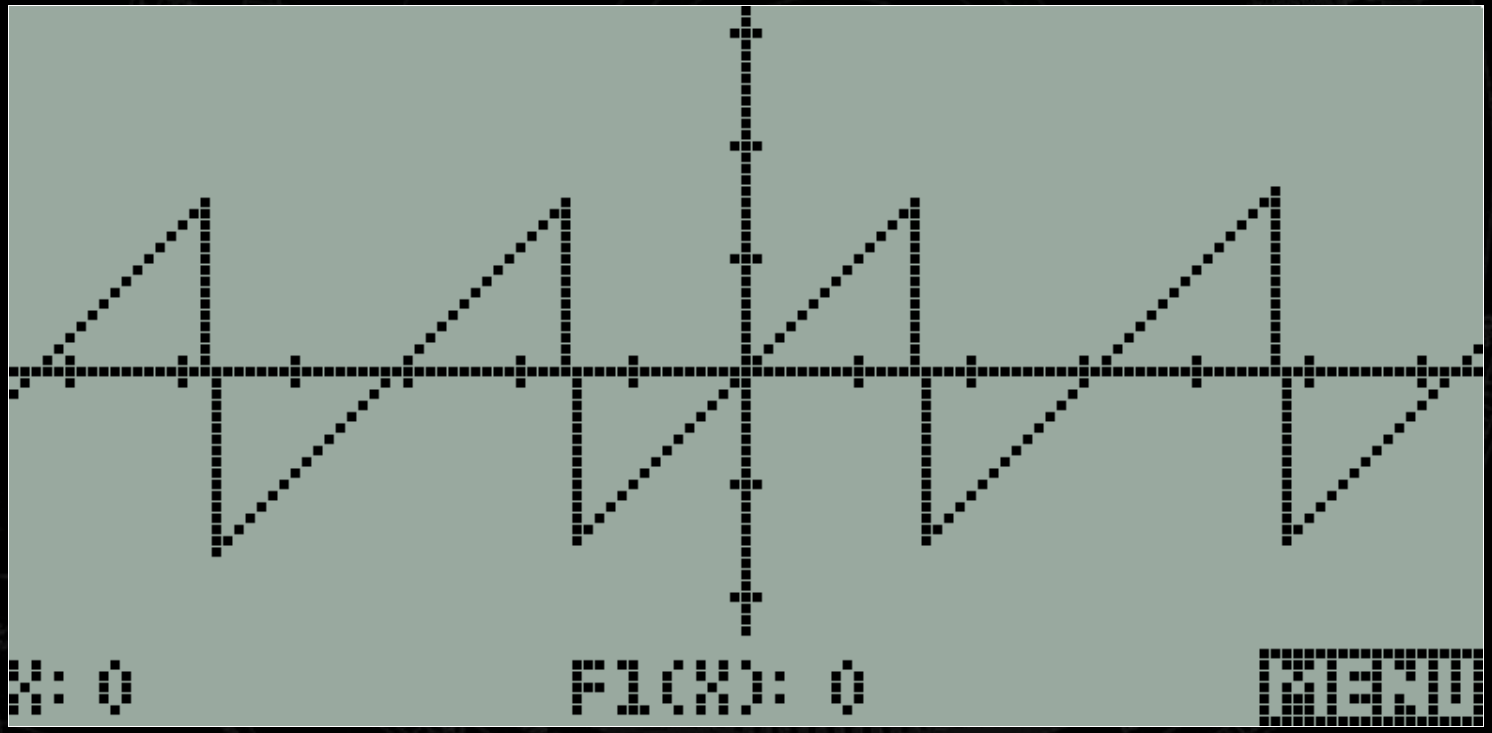
Mathematica 4 ImplicitPlot Output



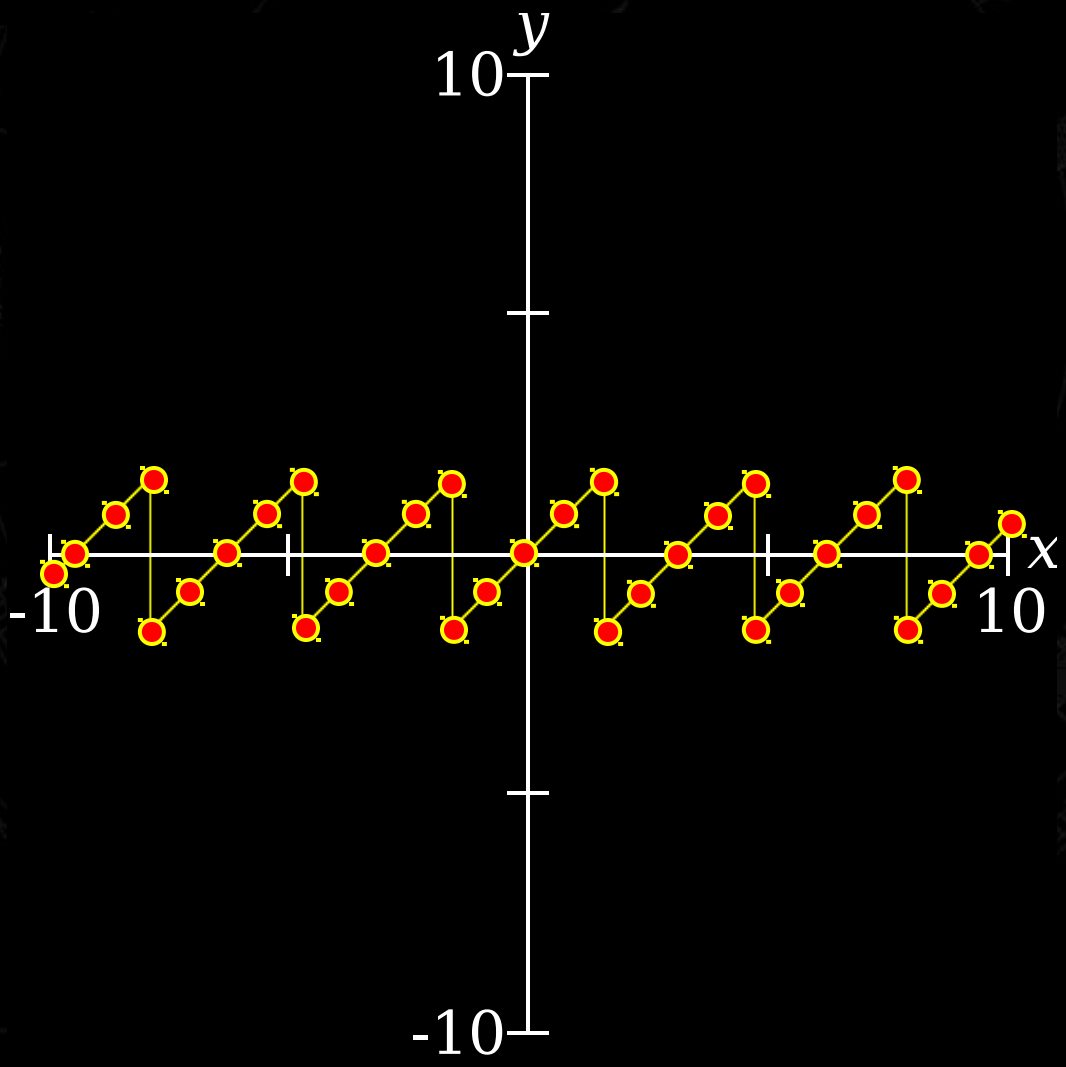
Texas Instruments TI-83 Plus



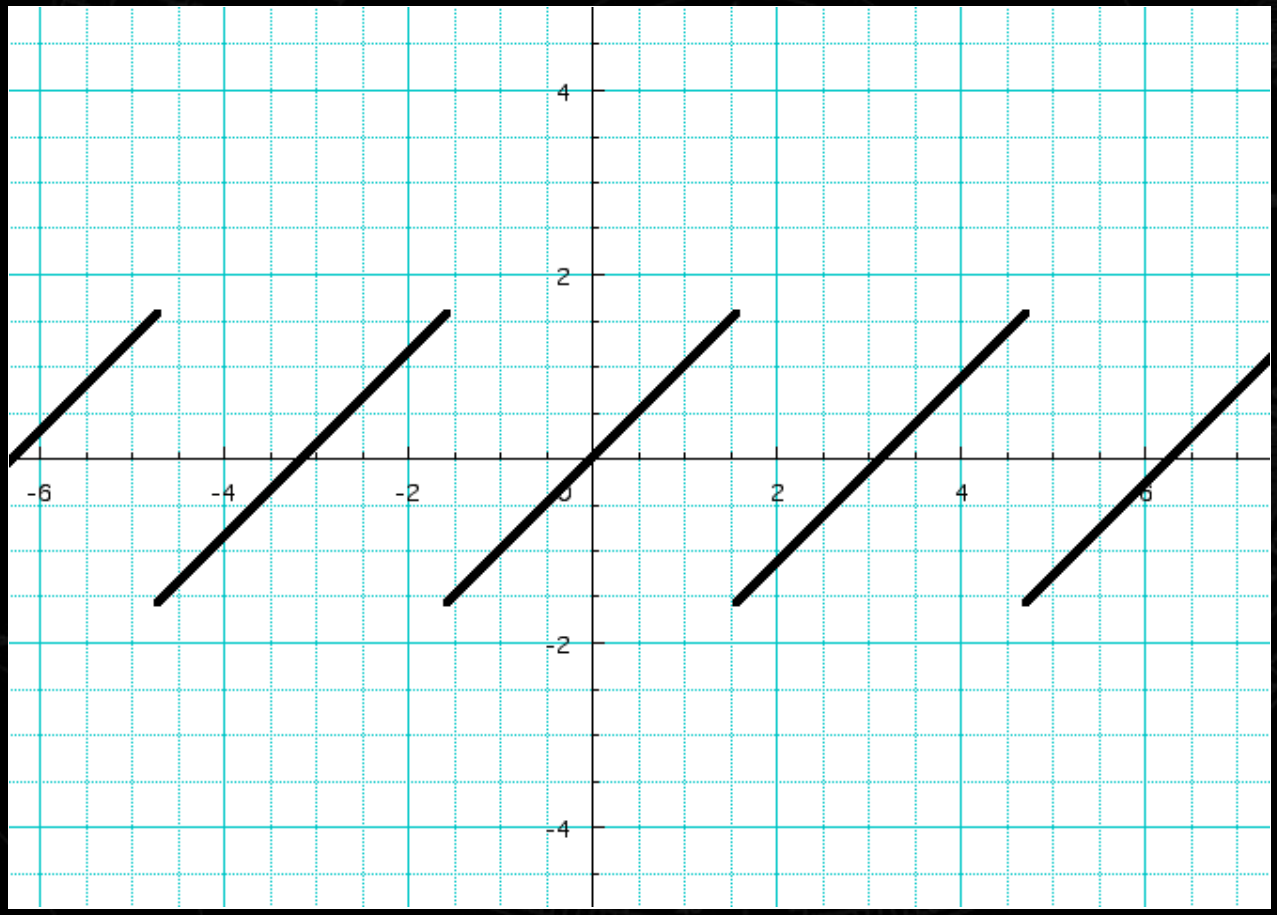
Hewlett-Packard HP 39G



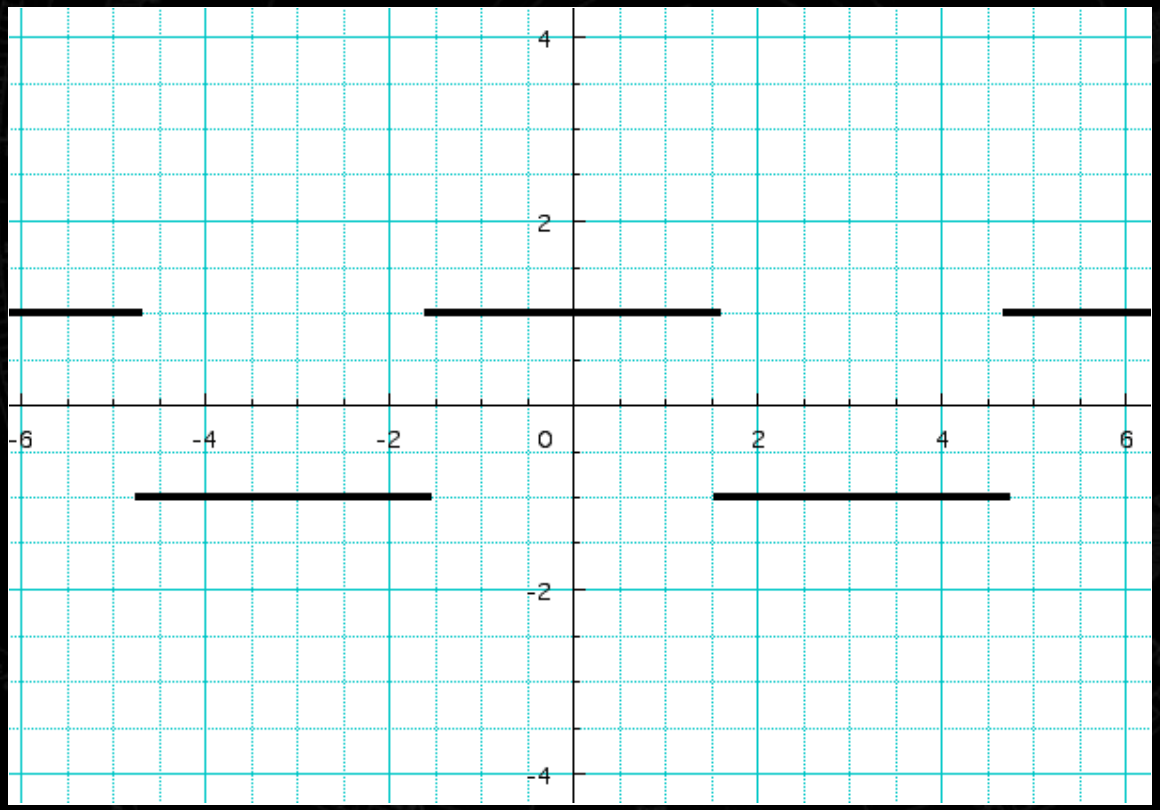
Connect-The-Dots Graphing



Curvus Pro 3.0.1

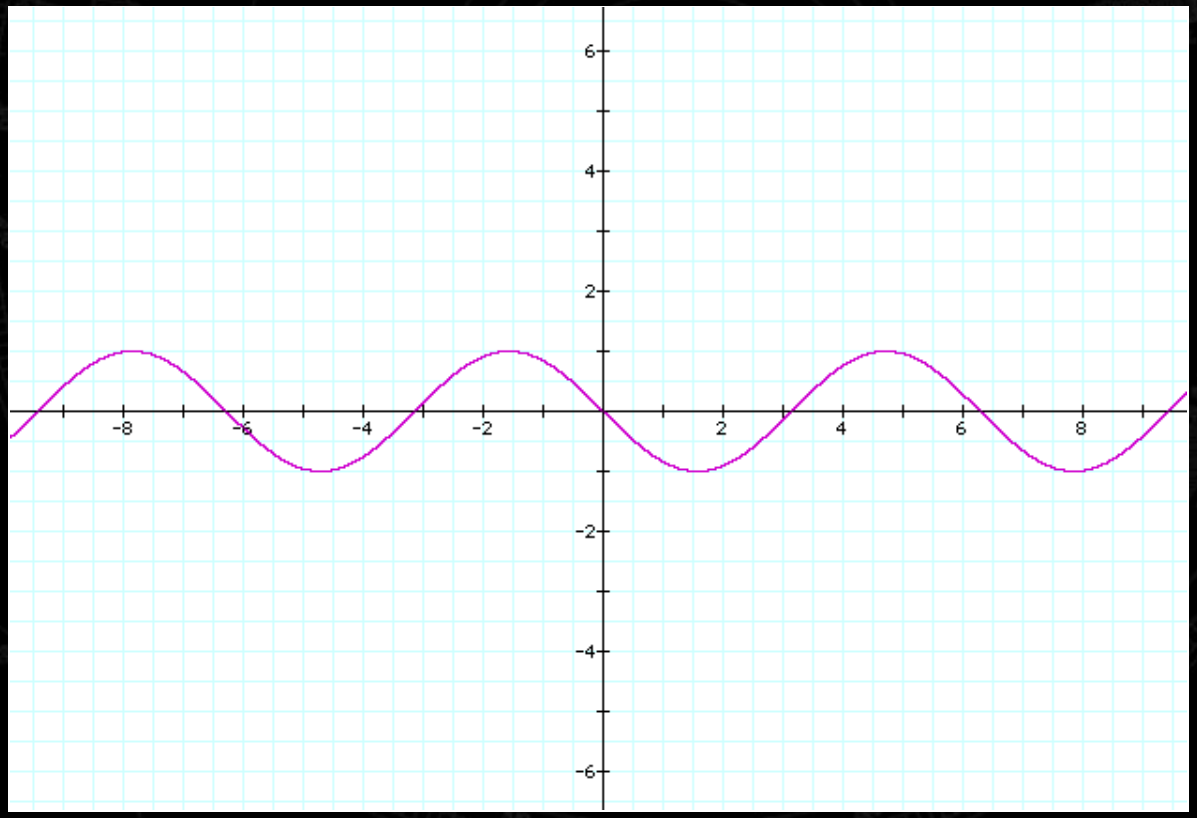


Curvus Pro 3.0.1



Floating-Point Arithmetic

Graphing Calculator 3.0.1 [Avitzur]



Connect-the-Dots Graphing

Problems:

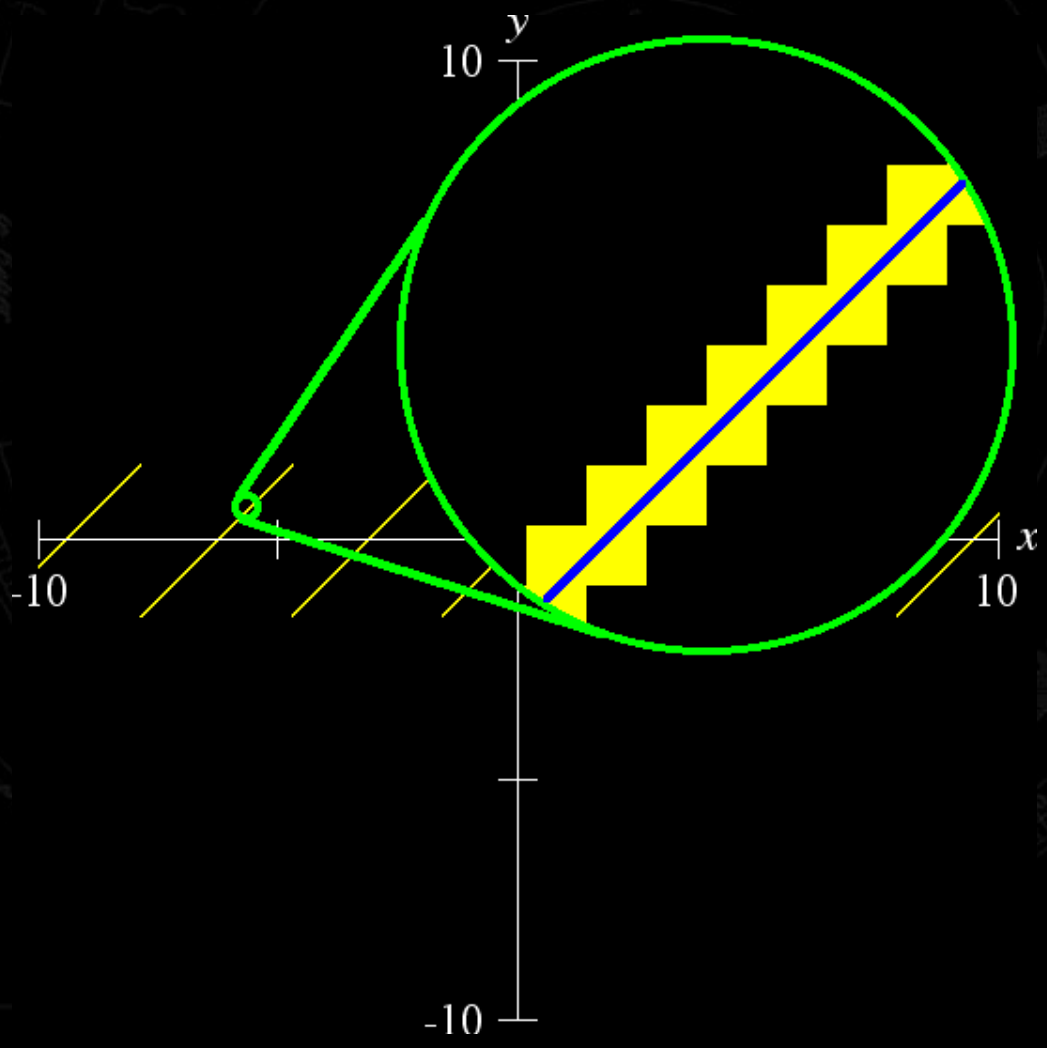
- **Not all dots should be connected**
- **Dots may be far from the curve**

Connect-the-Dots Graphing

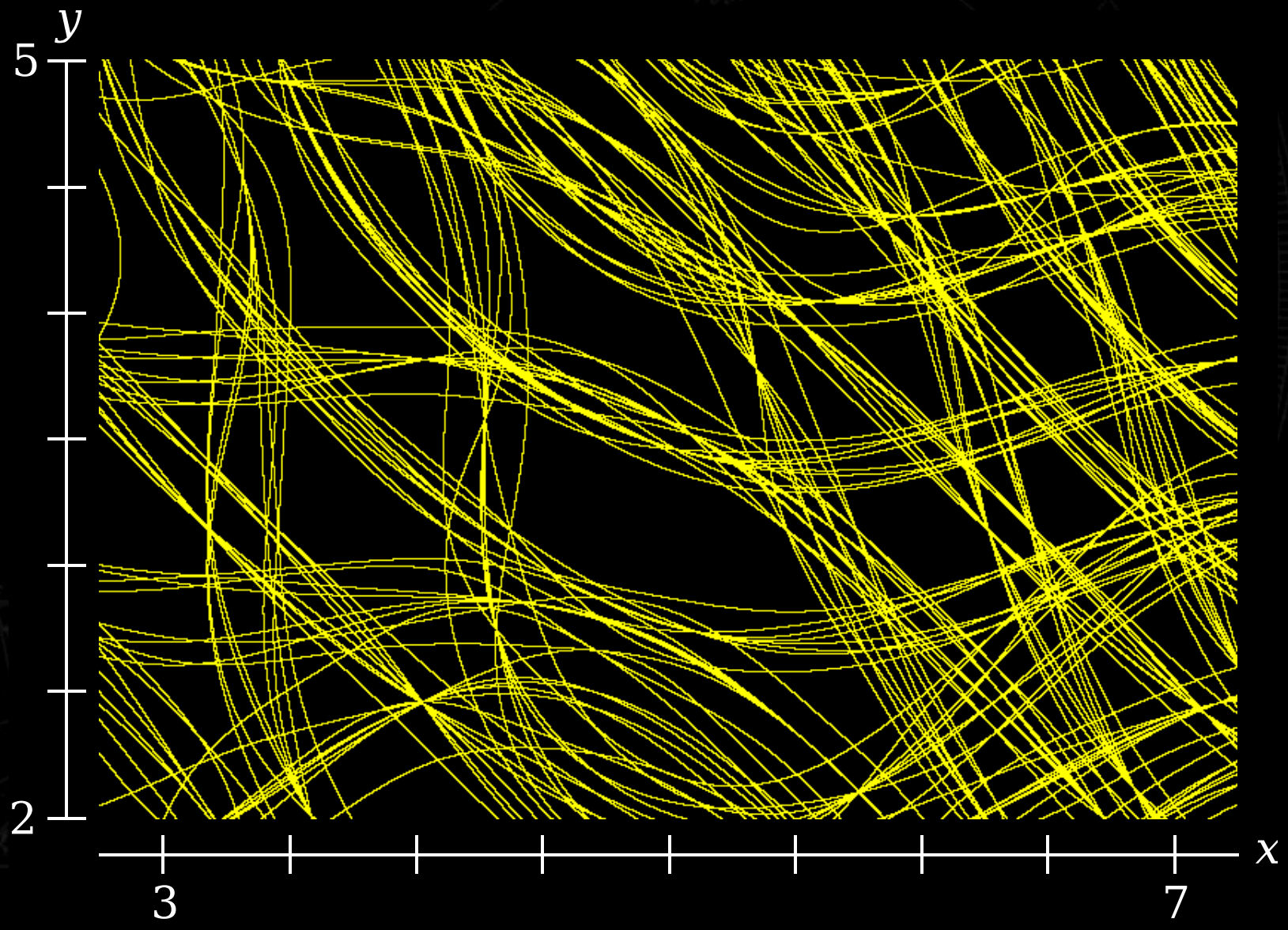
Fundamental Problem:

- **We haven’t defined the graph’s semantics**

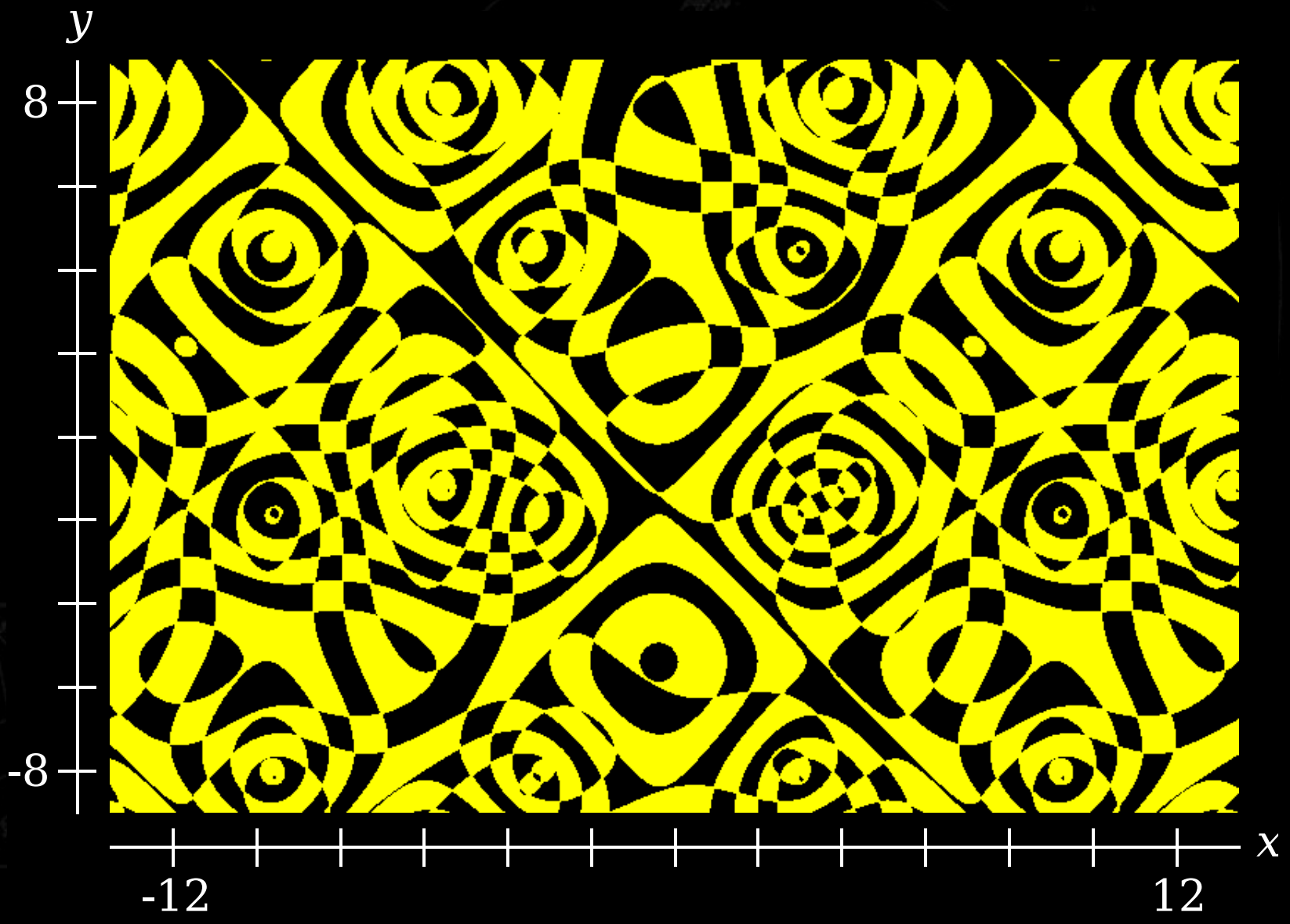
Graph Semantics



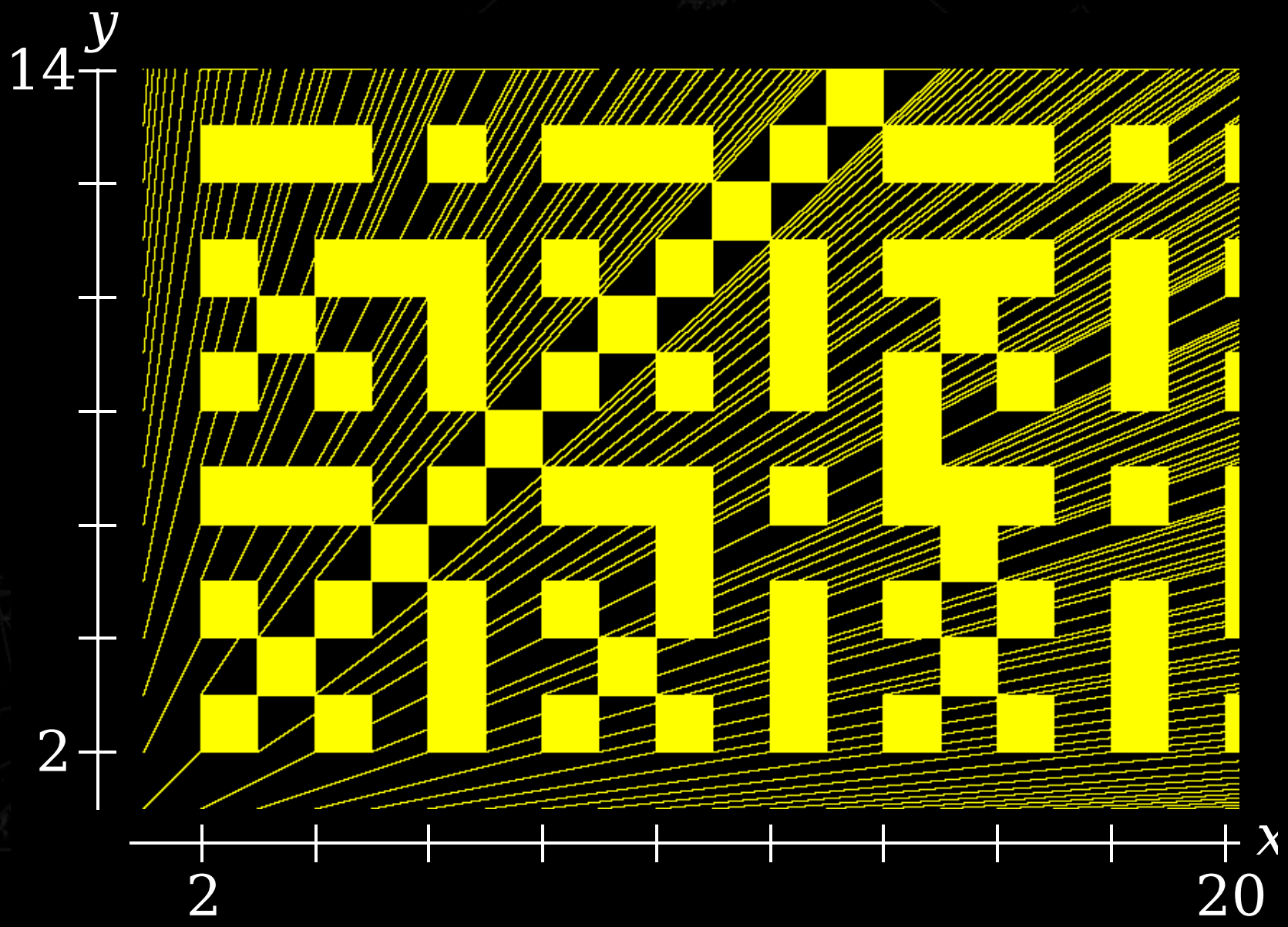
Example Graph



Example Graph



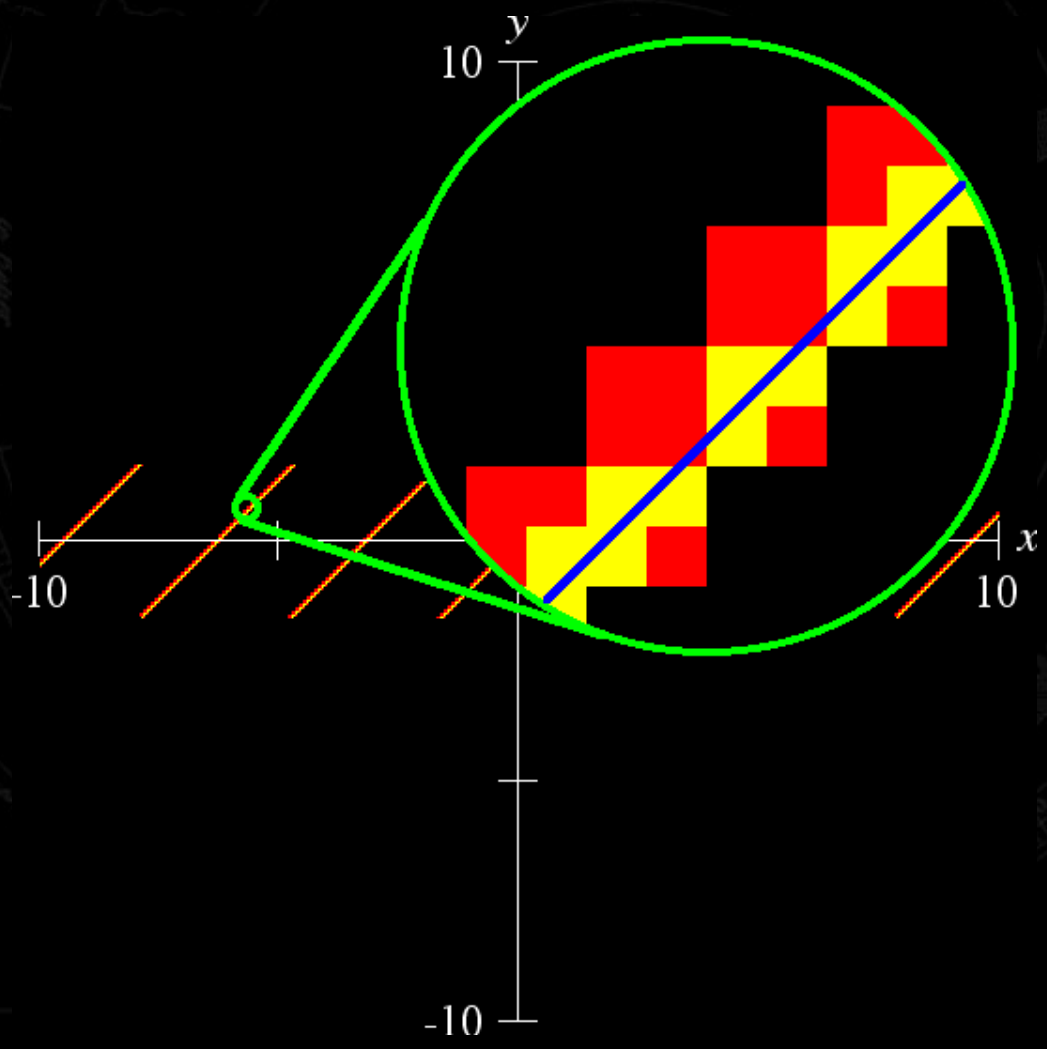
Example Graph



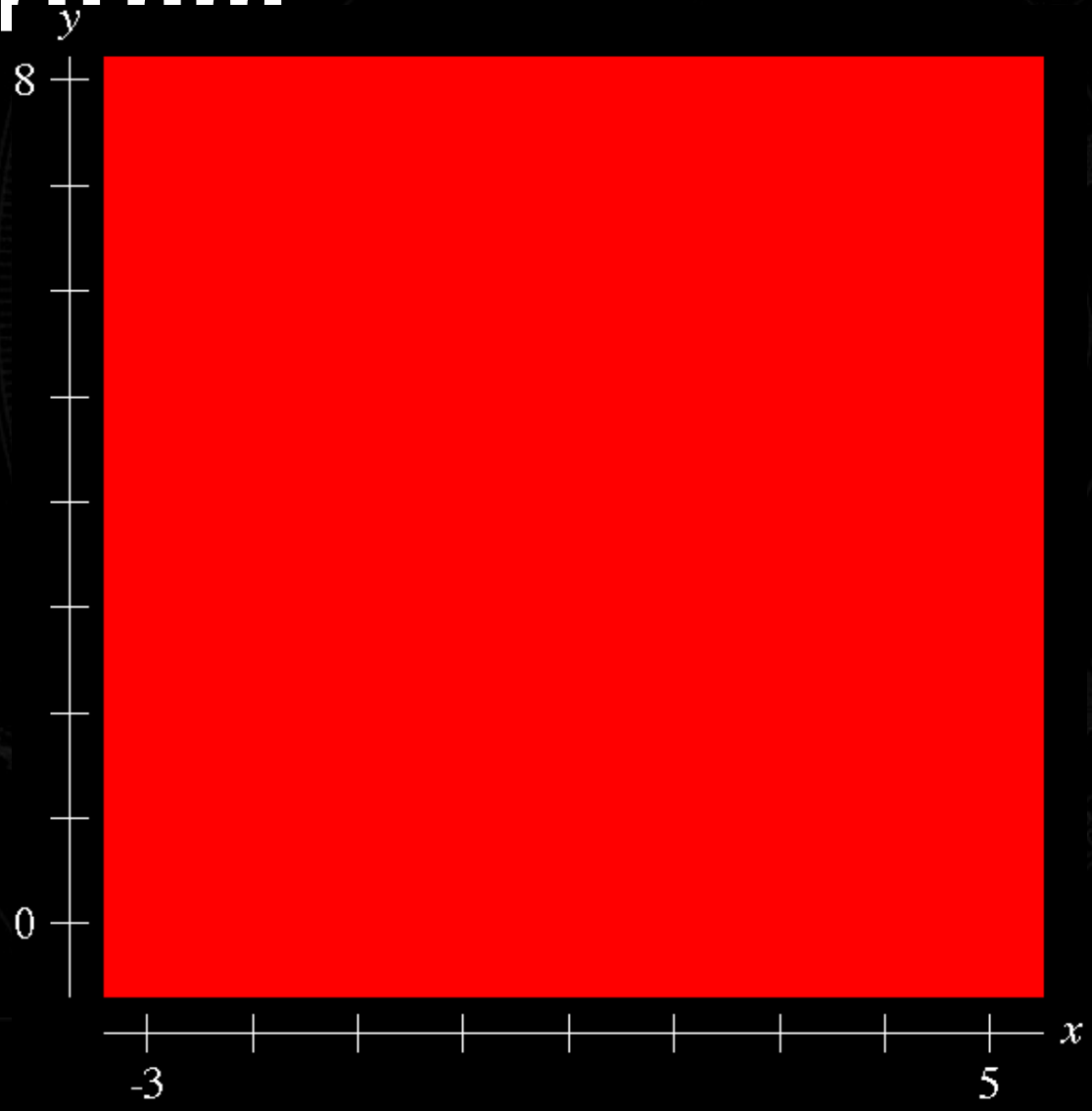
Unfortunate Reality

- This naïve goal is impossible since graphing, as formalized, is not computable

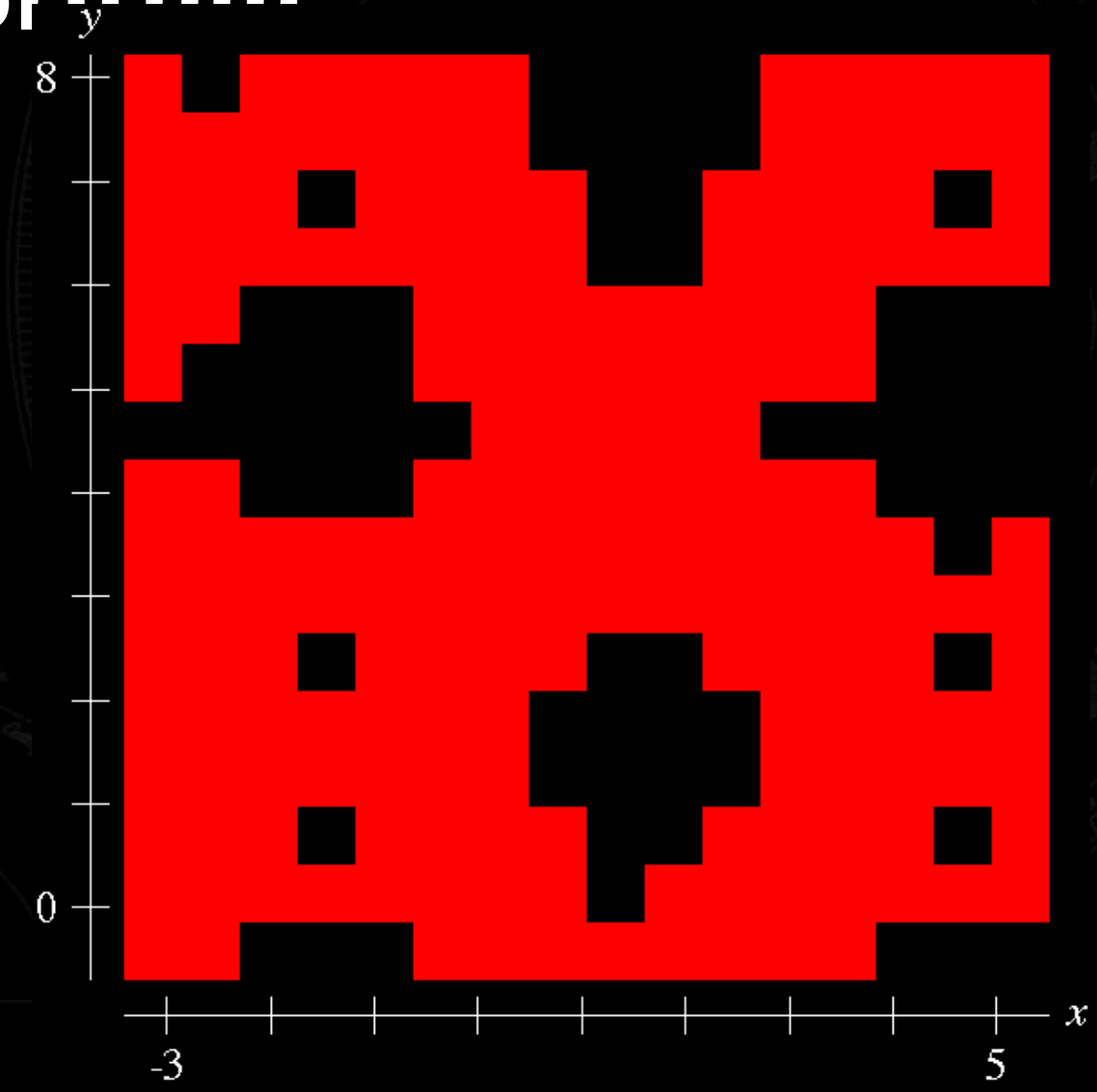
Practical Graph Semantics



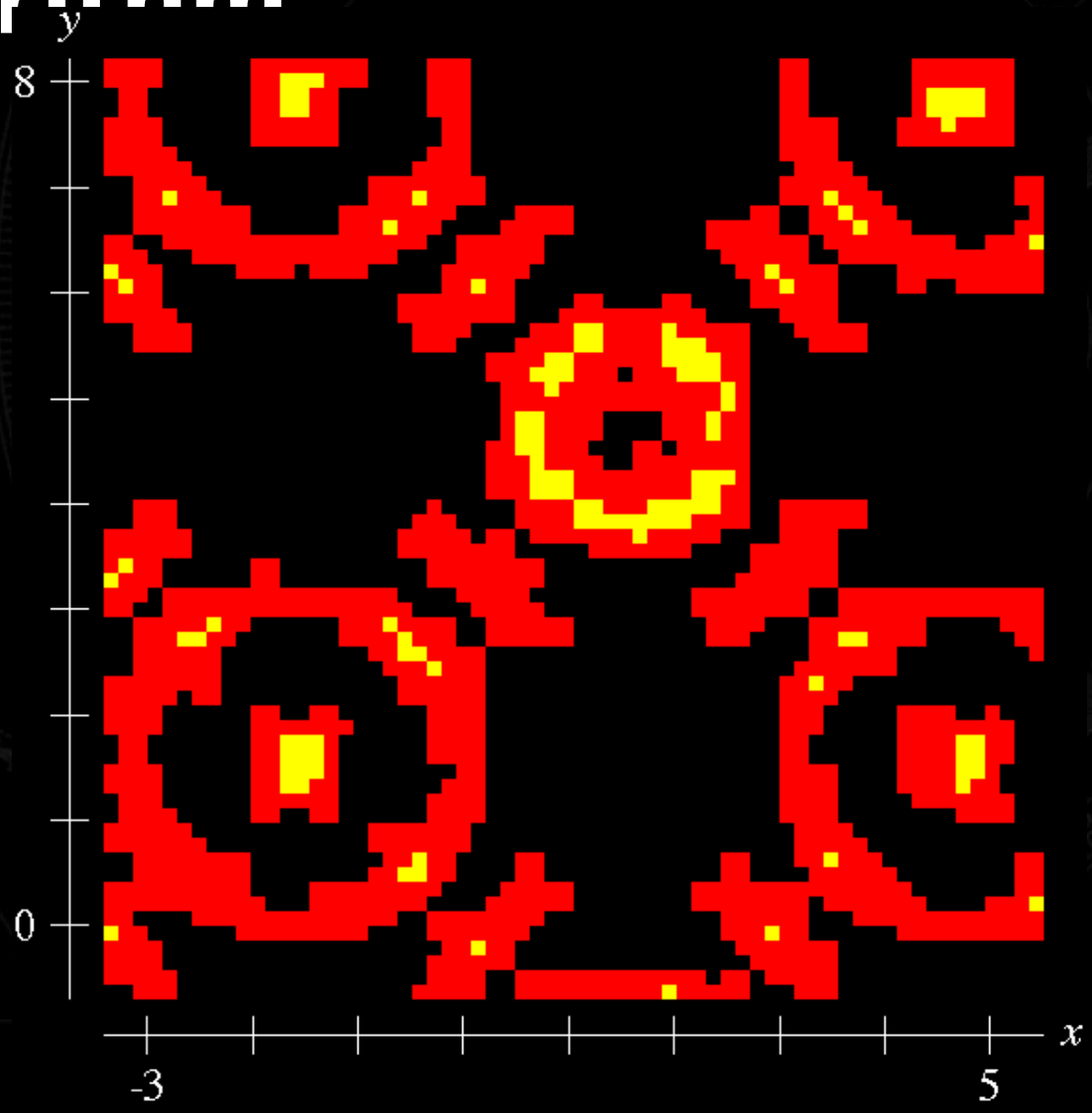
Iterative Graphing Algorithm



Iterative Graphing Algorithm



Iterative Graphing Algorithm



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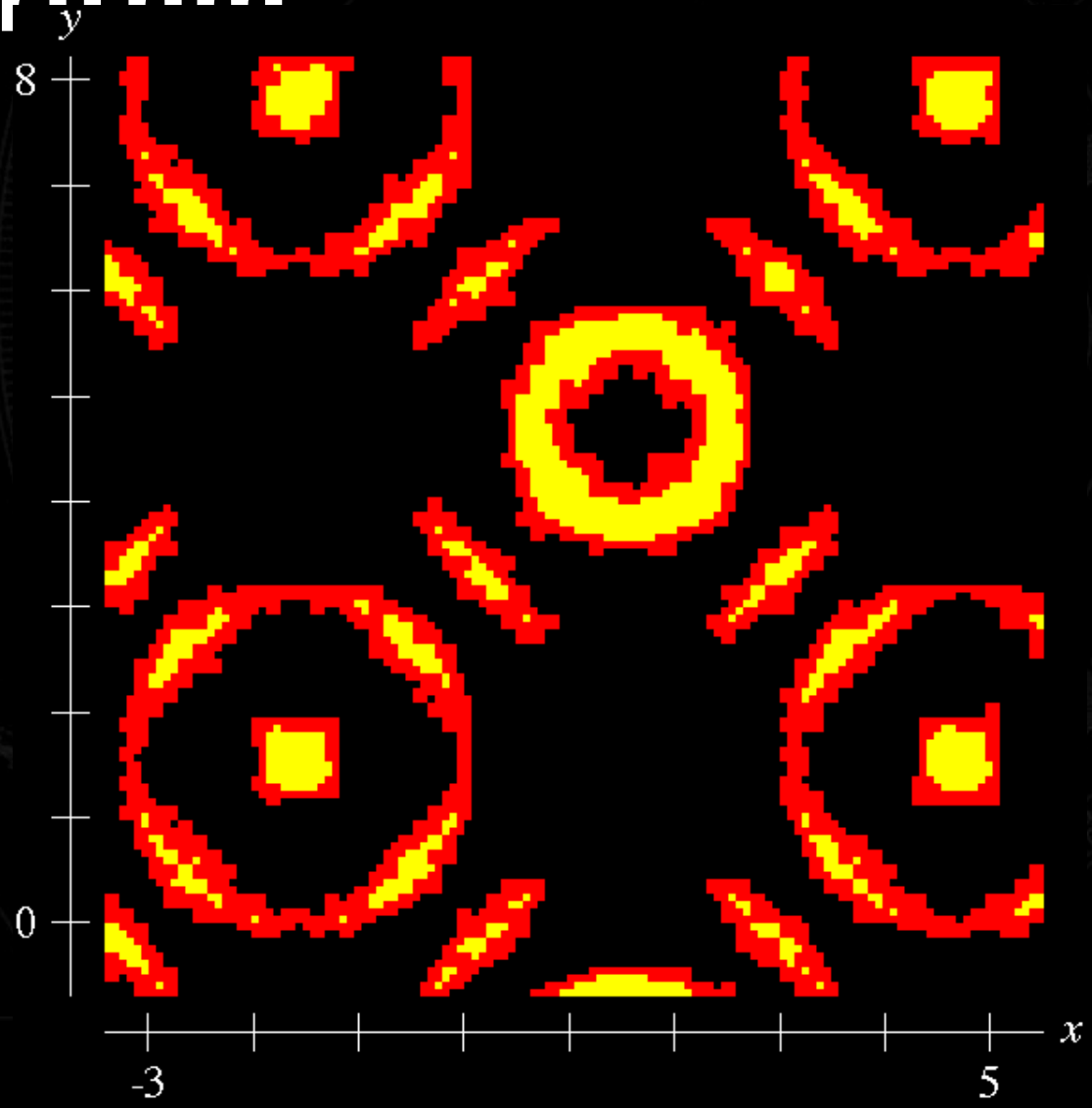
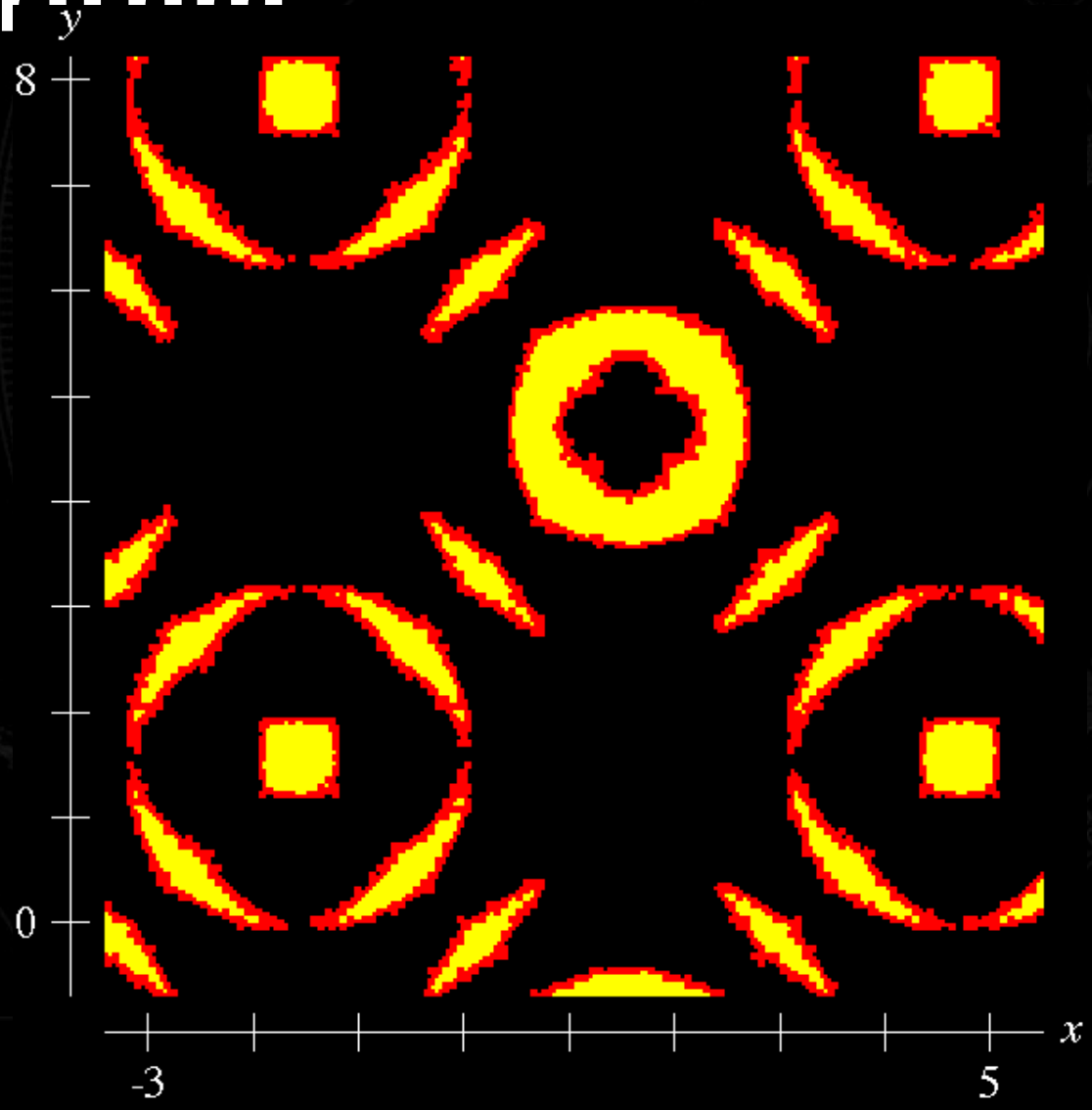
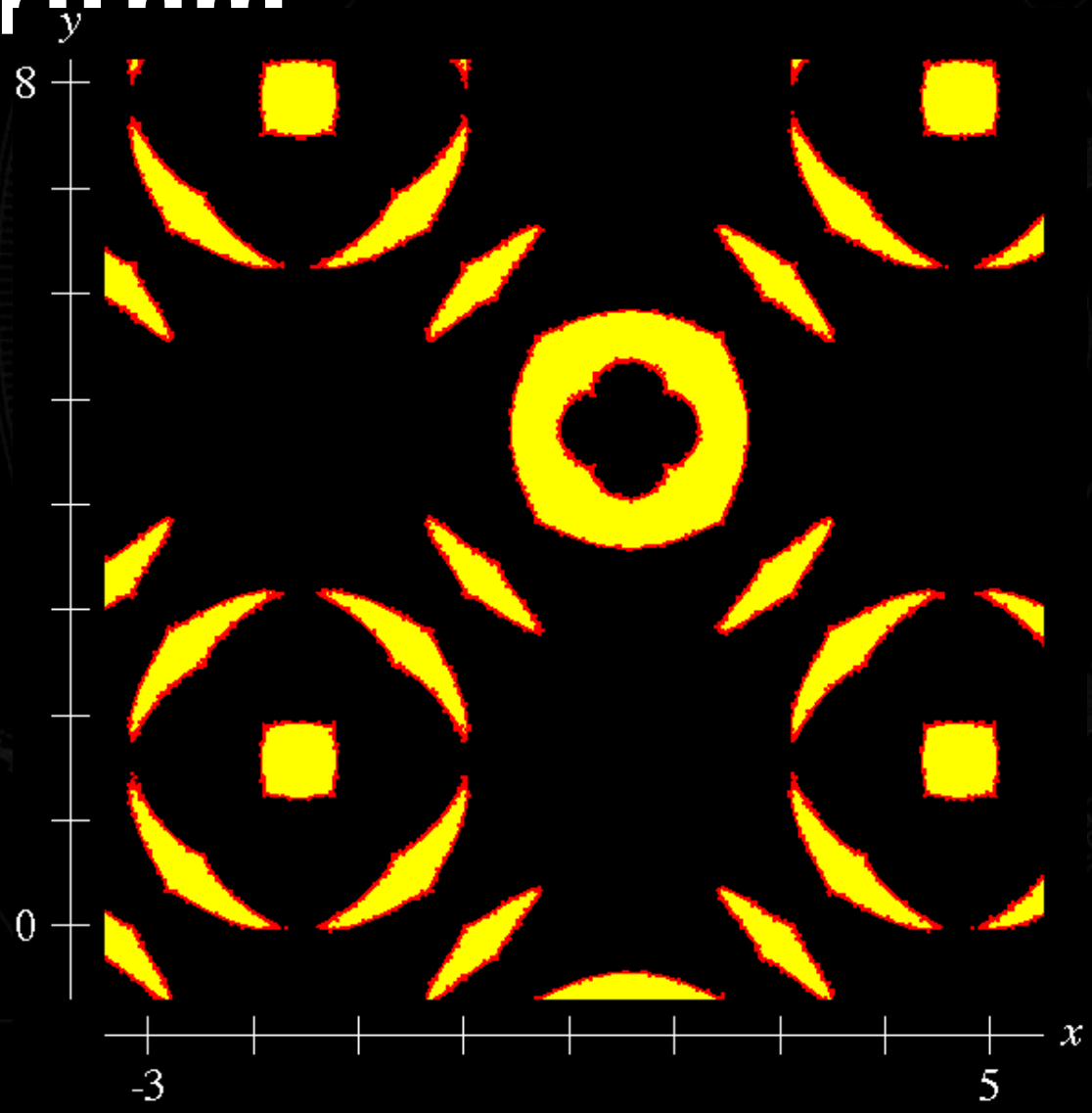


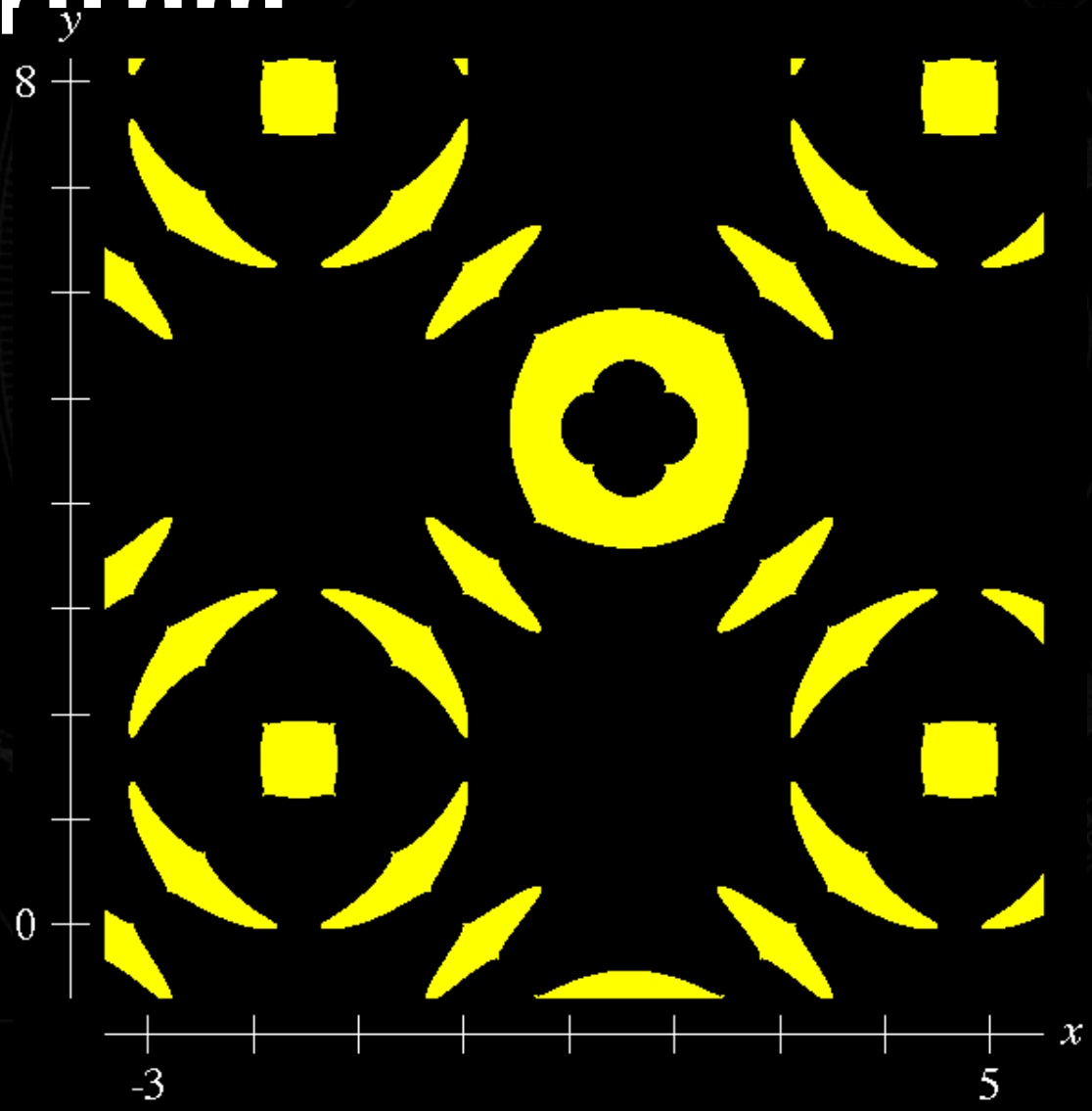
Iterative Graphing Algorithm



Iterative Graphing Algorithm



Iterative Graphing Algorithm



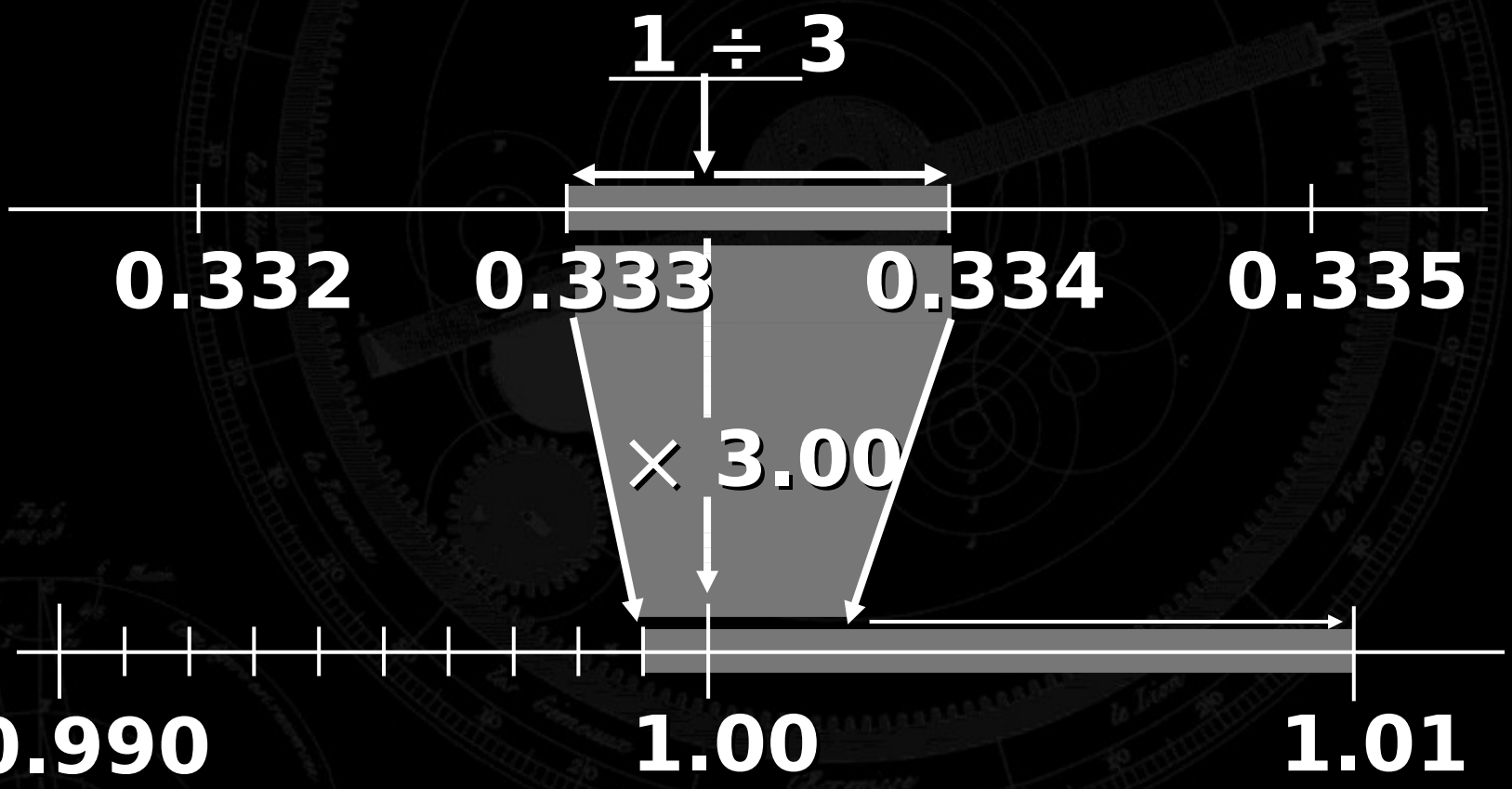
Reliable Graphing

- We now have a well-defined problem
- But how do we evaluate formulae?

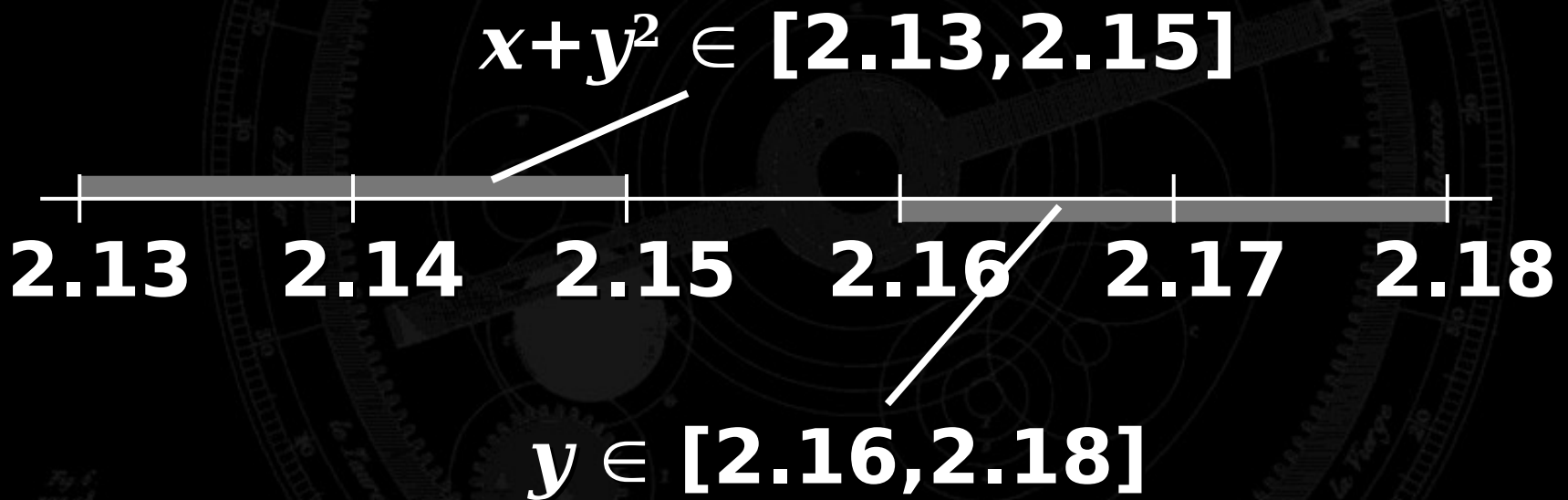
Formula Evaluation

- Use interval arithmetic to evaluate formulae
- Interval arithmetic provides guaranteed bounds on accuracy

Interval Arithmetic

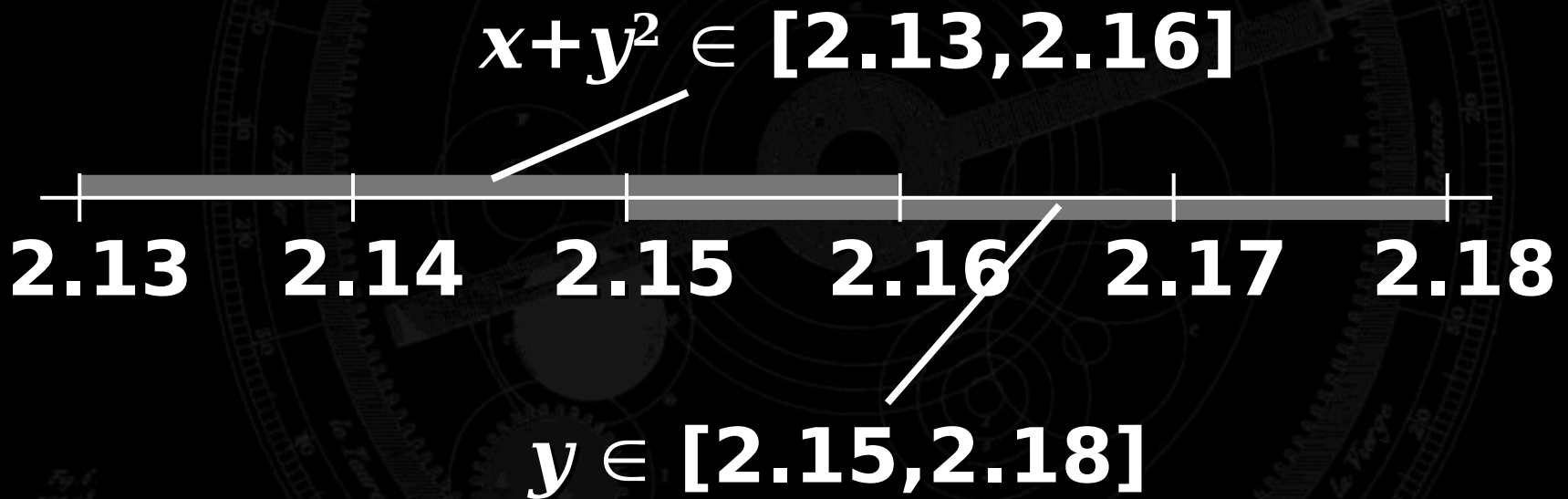


Interval Comparisons



Is $x+y^2<y$? Yes.

Interval Comparisons



Is $x+y^2<y$? Maybe.

Domain Tracking

Is $\sqrt{x} < y$? Yes.

Domain Tracking

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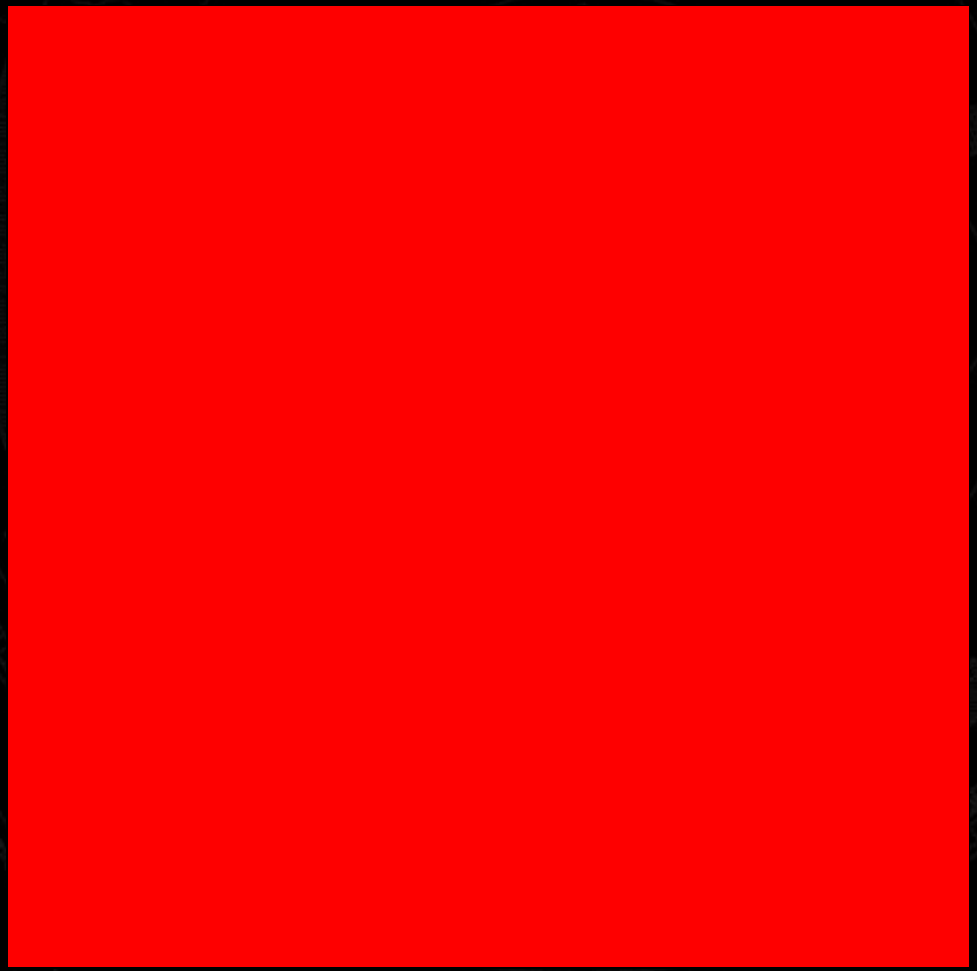
Algorithm A



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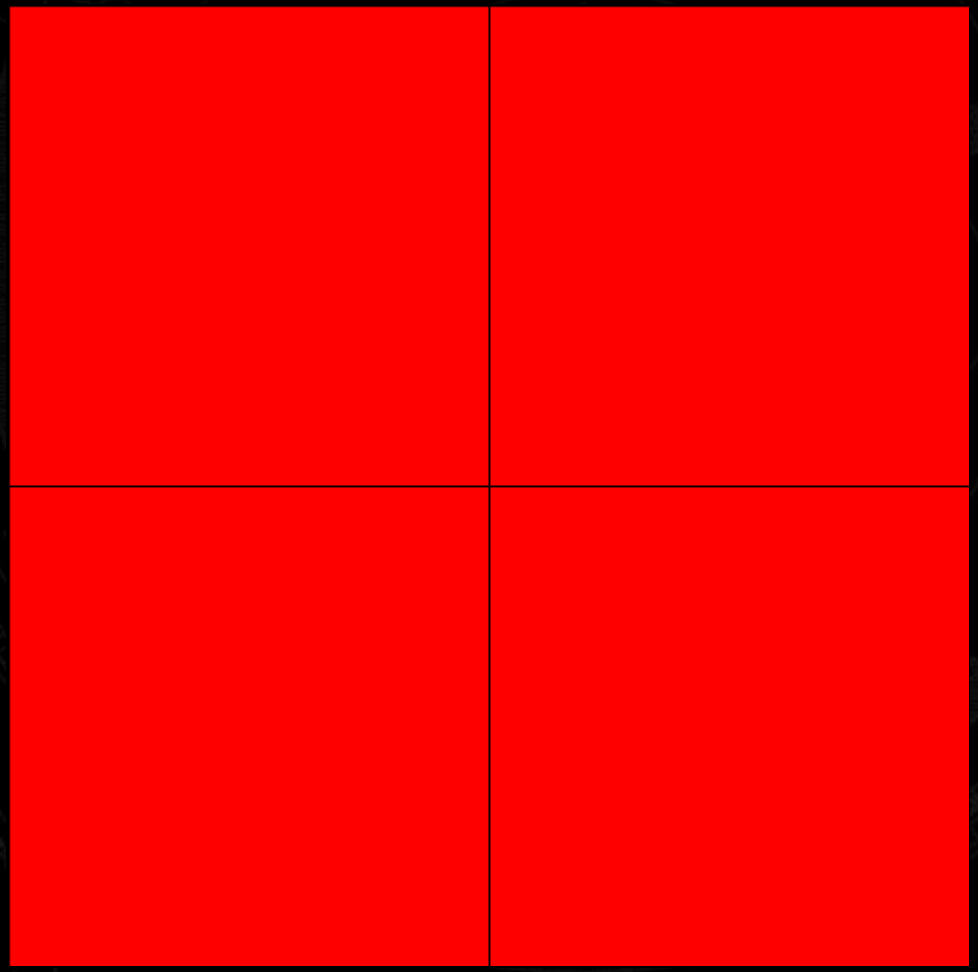

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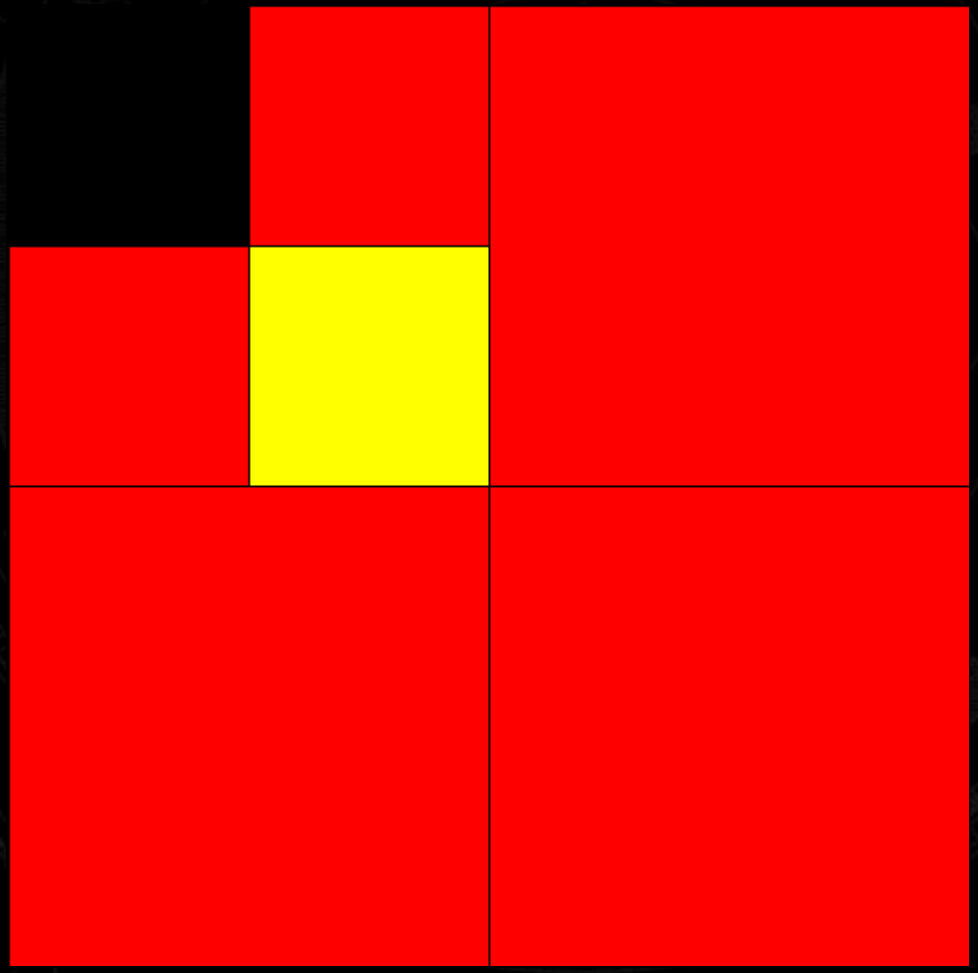

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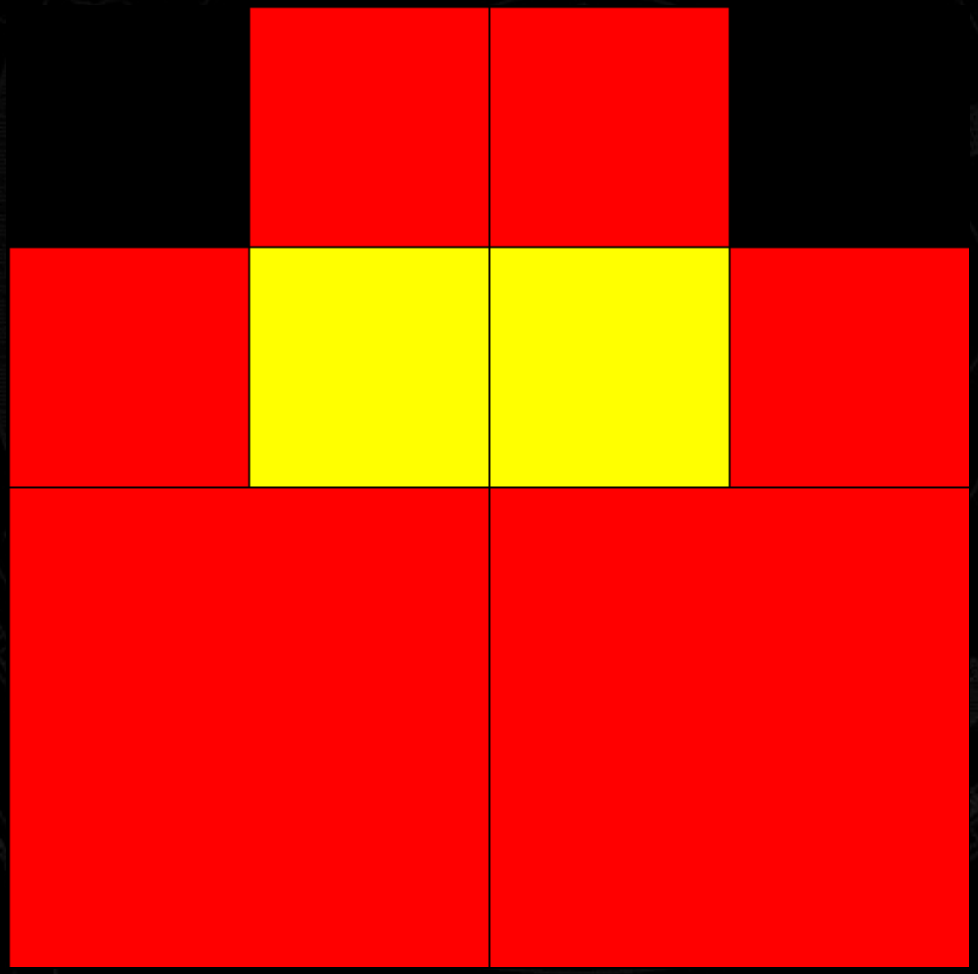

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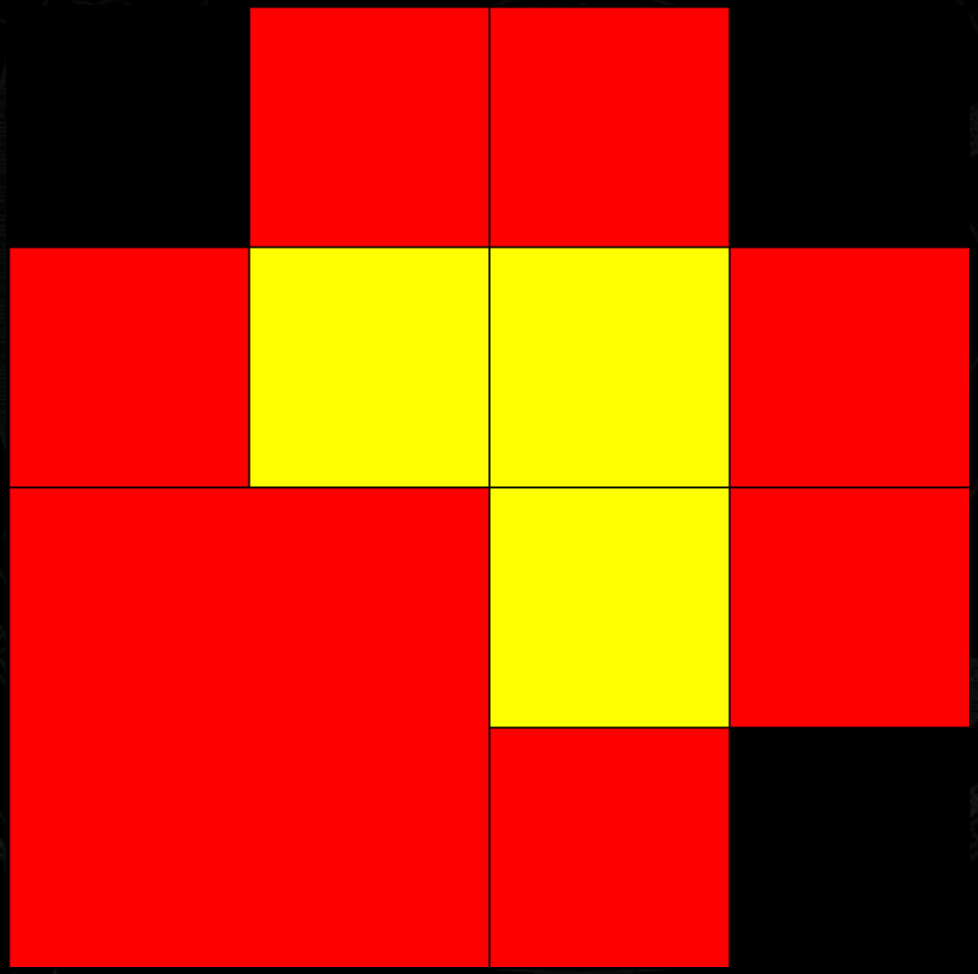

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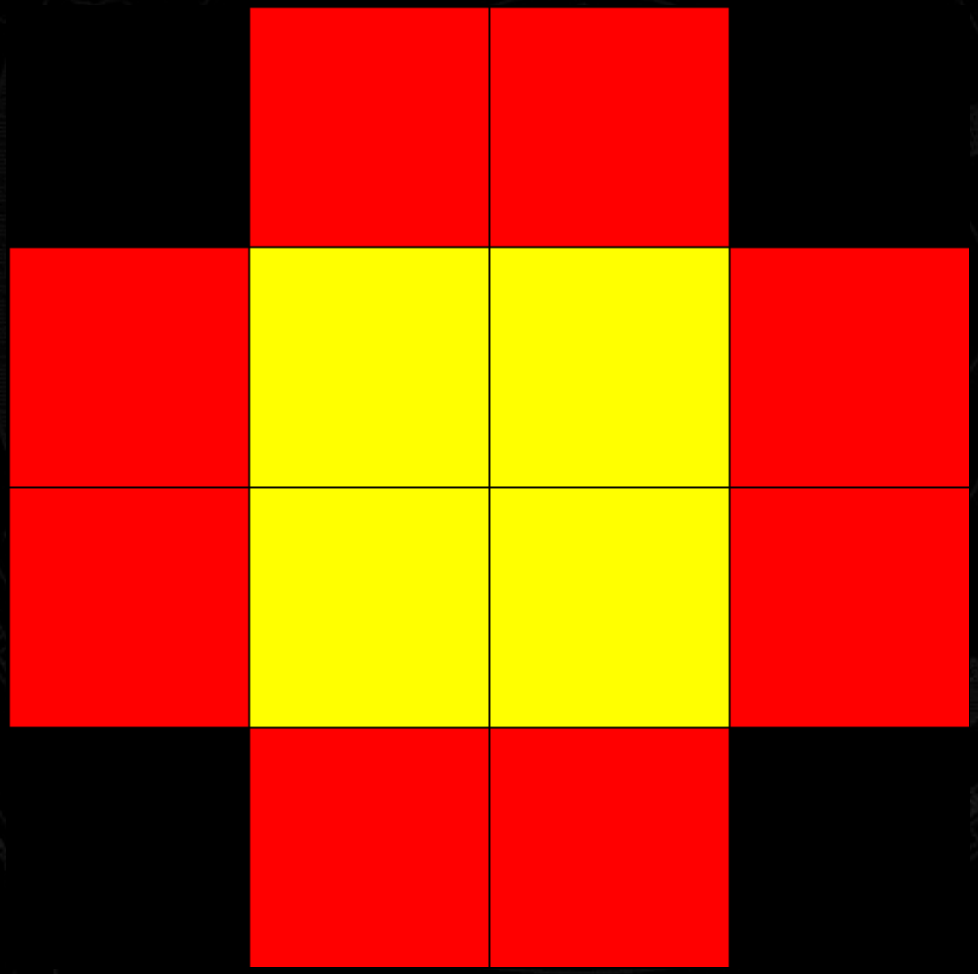

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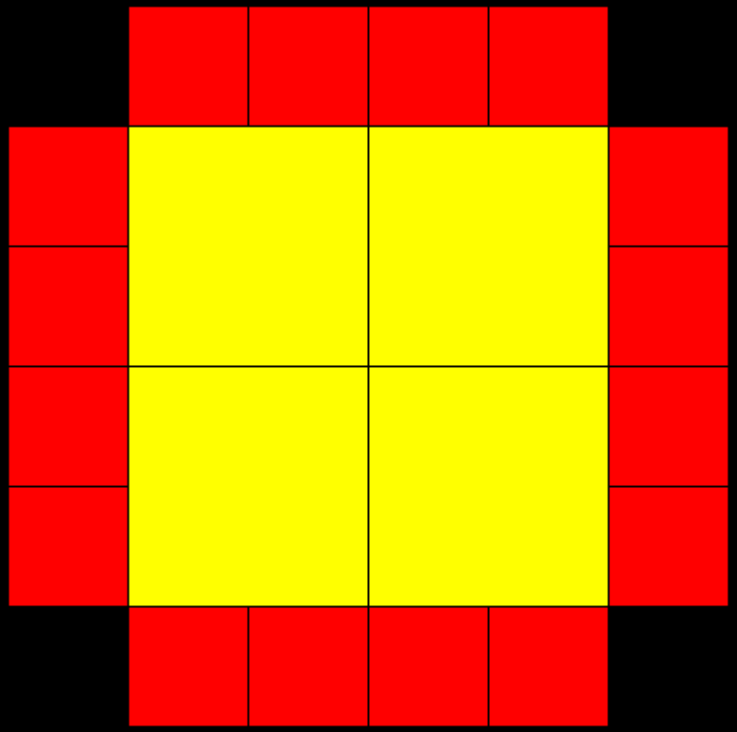

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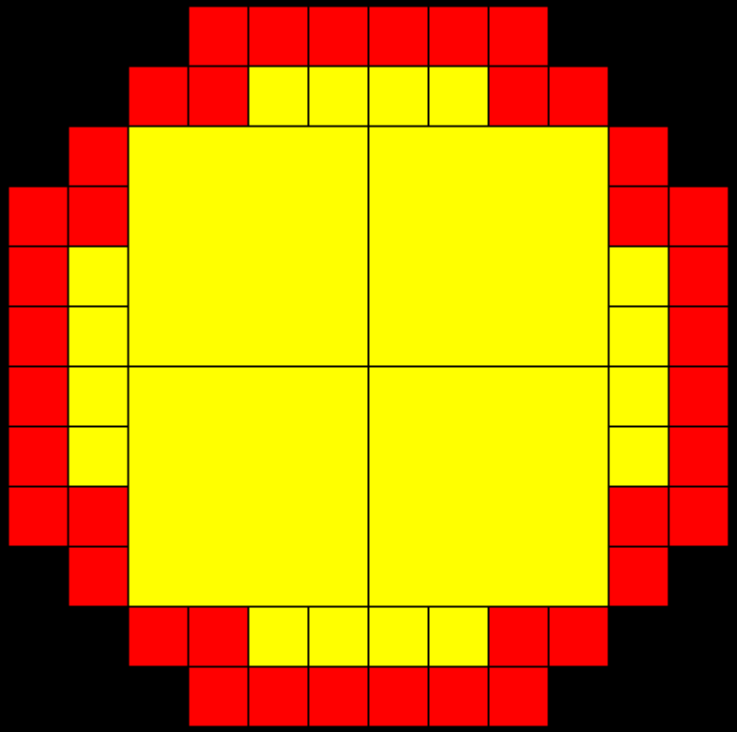

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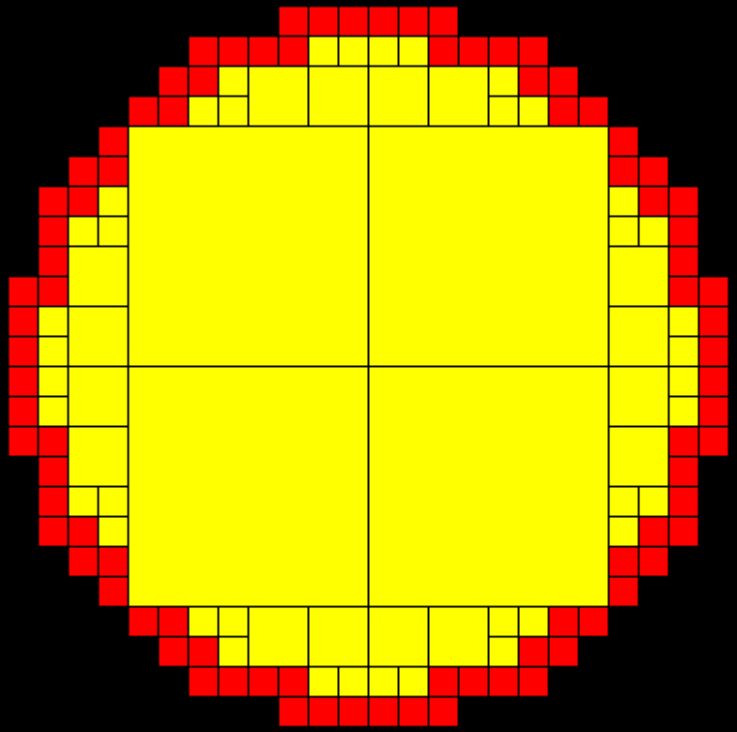

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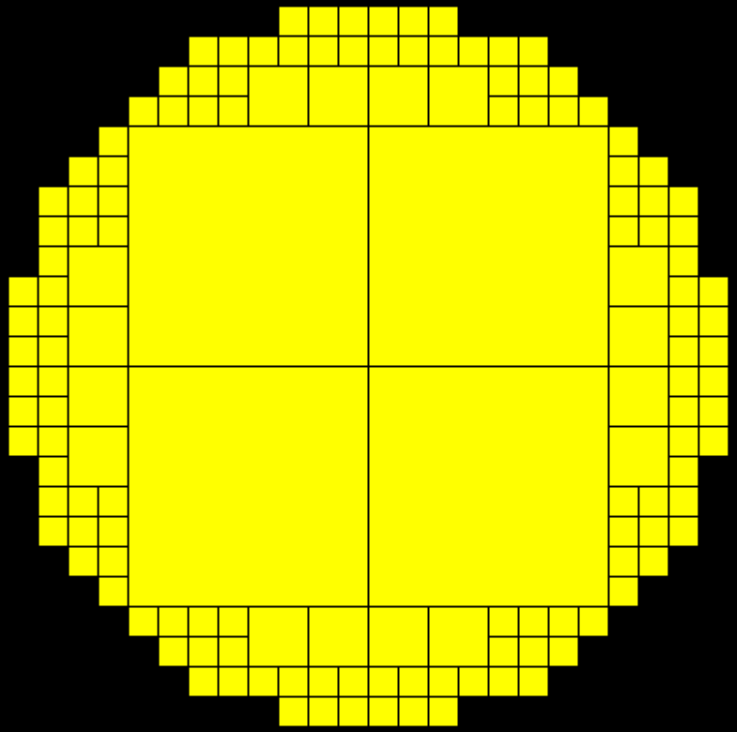
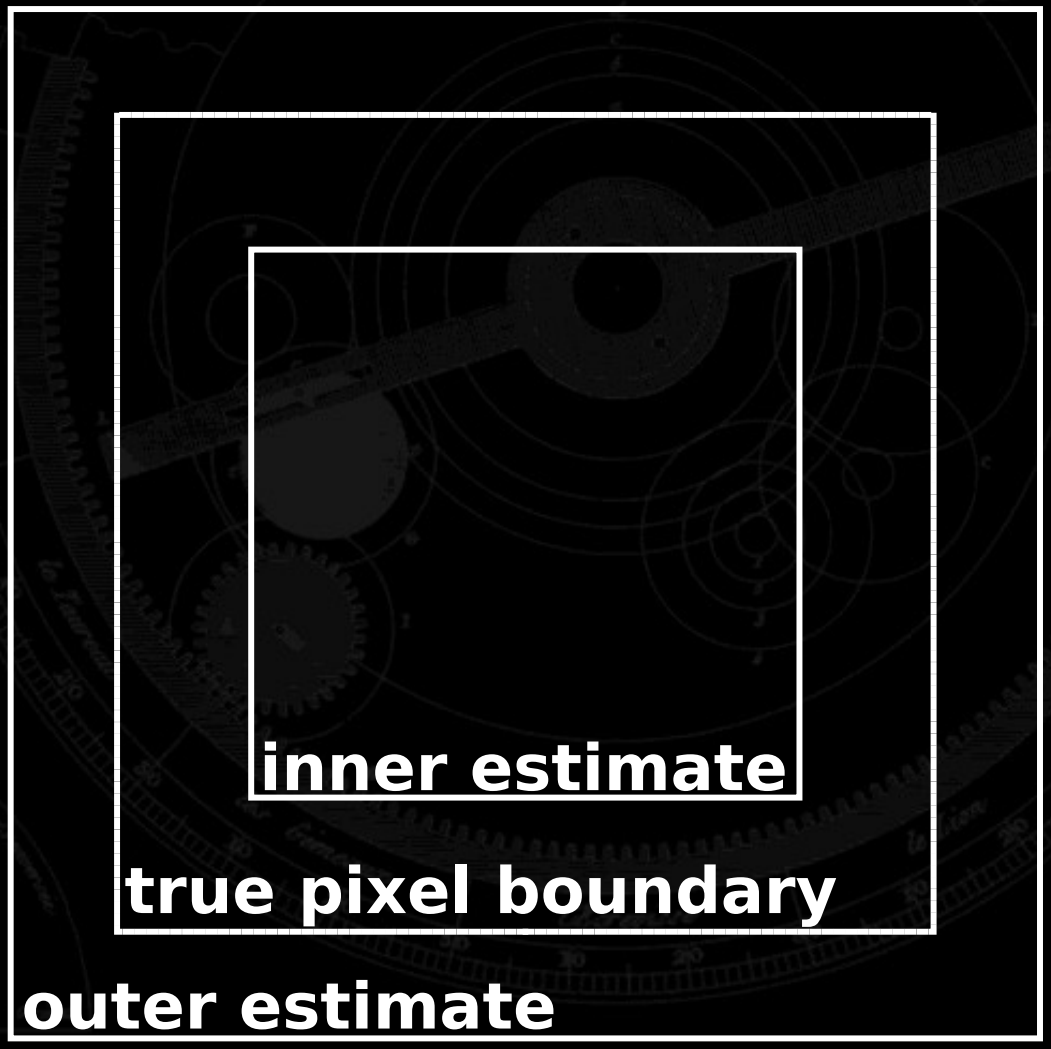
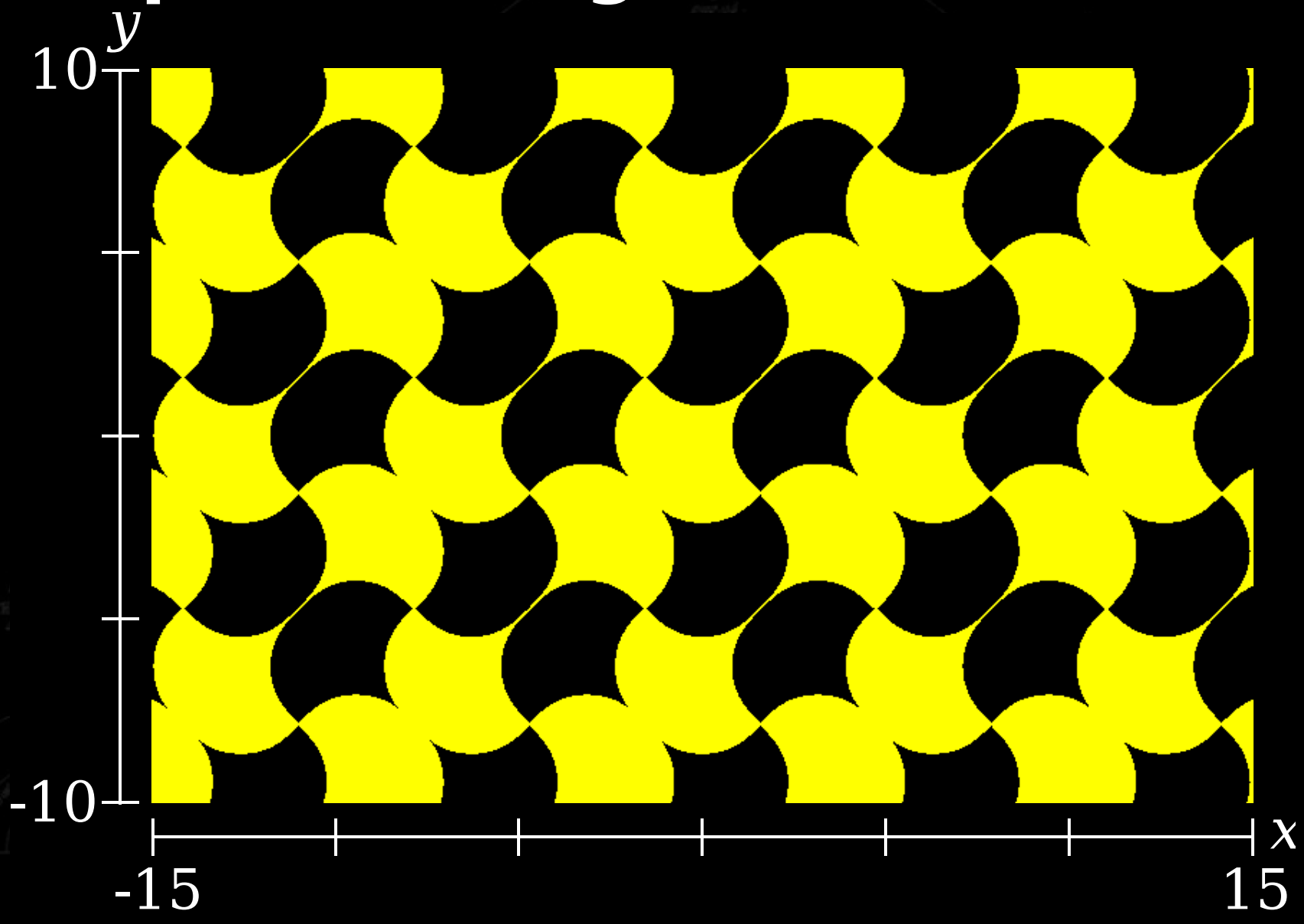


Pixel Boundaries



Graph from Algorithm A



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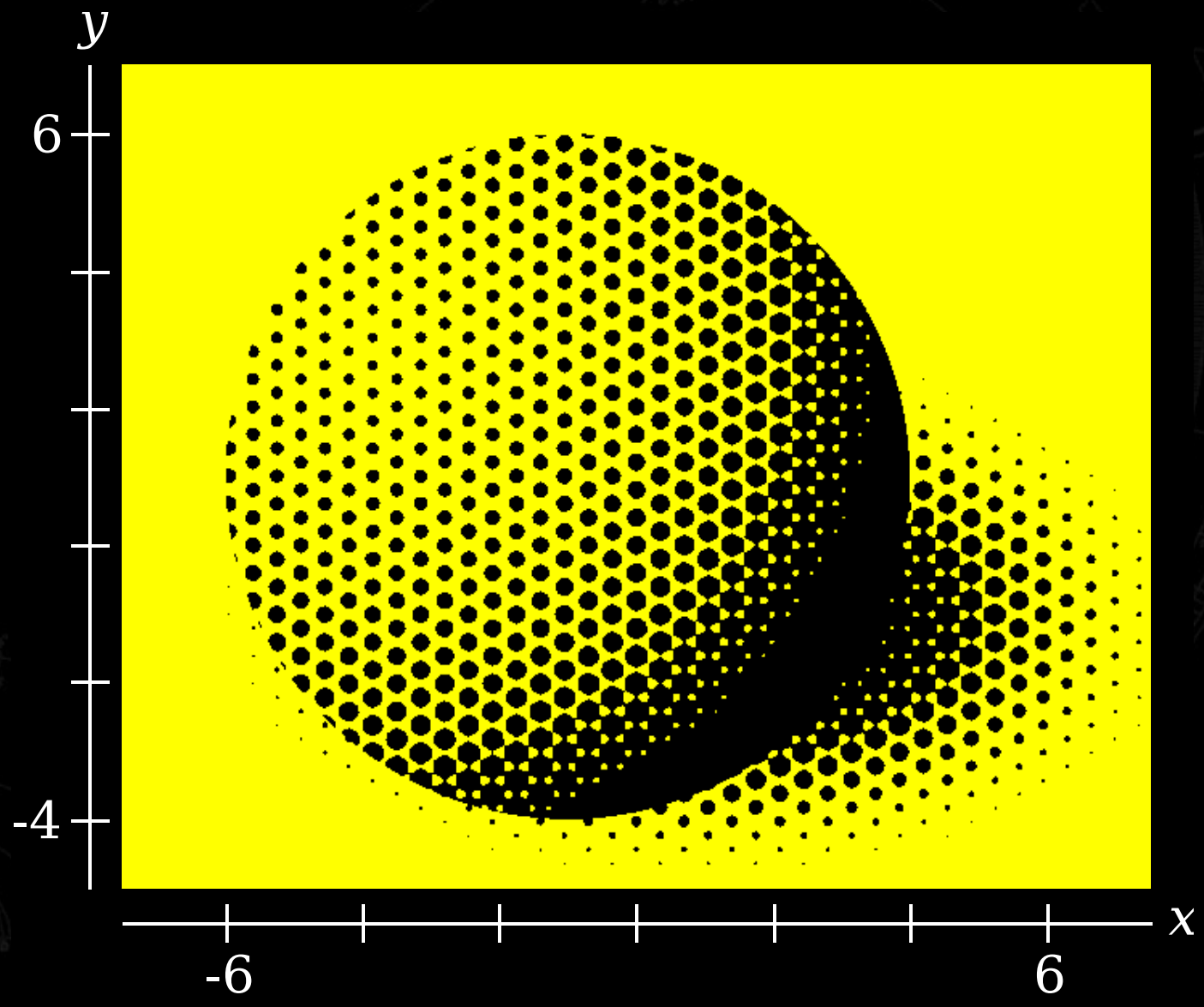

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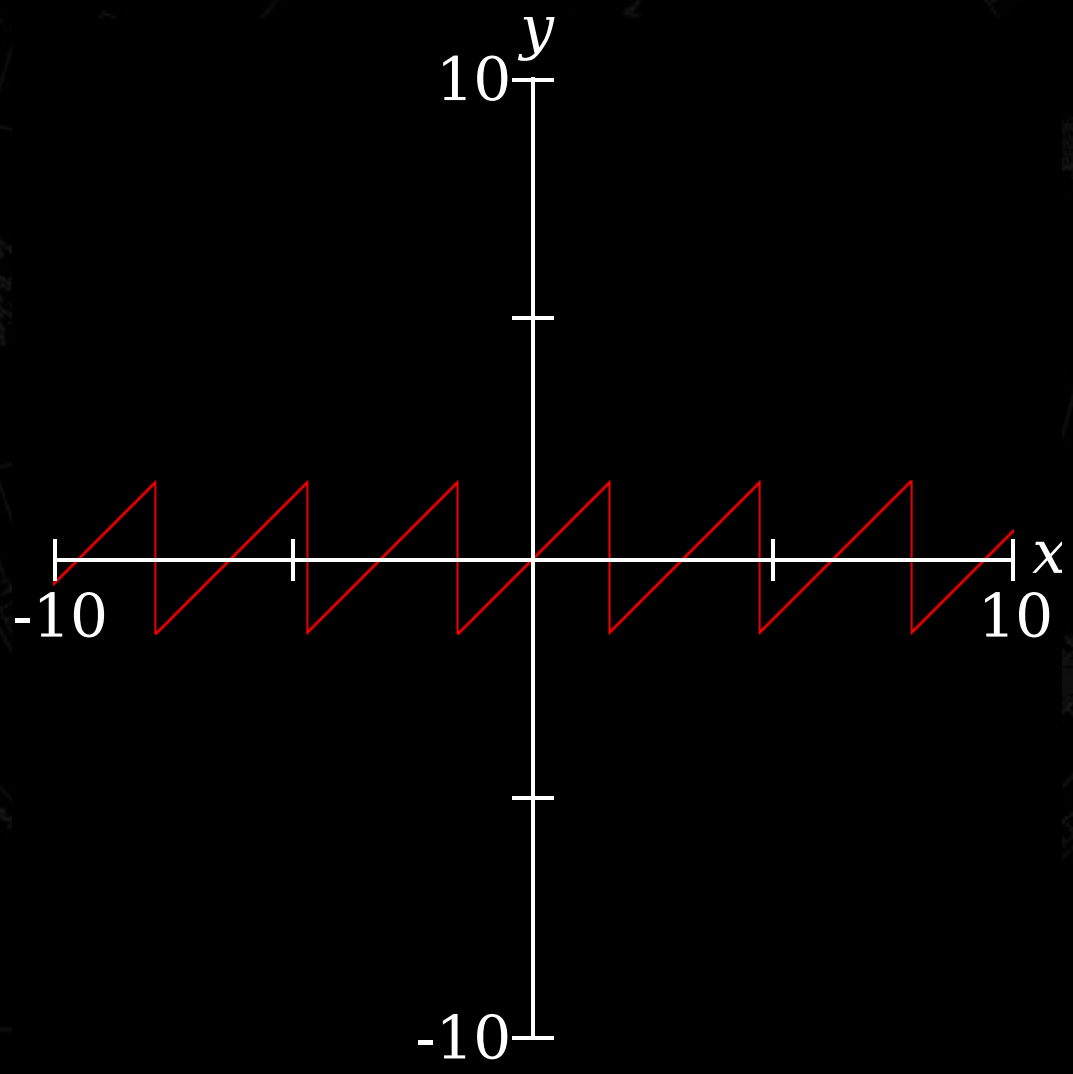
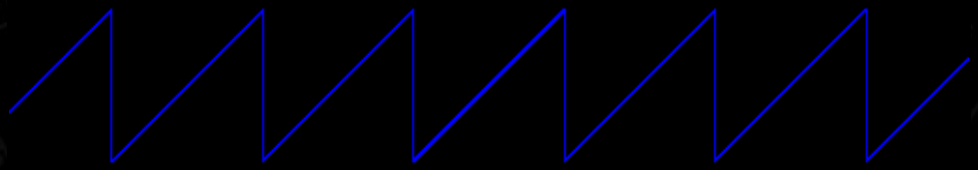
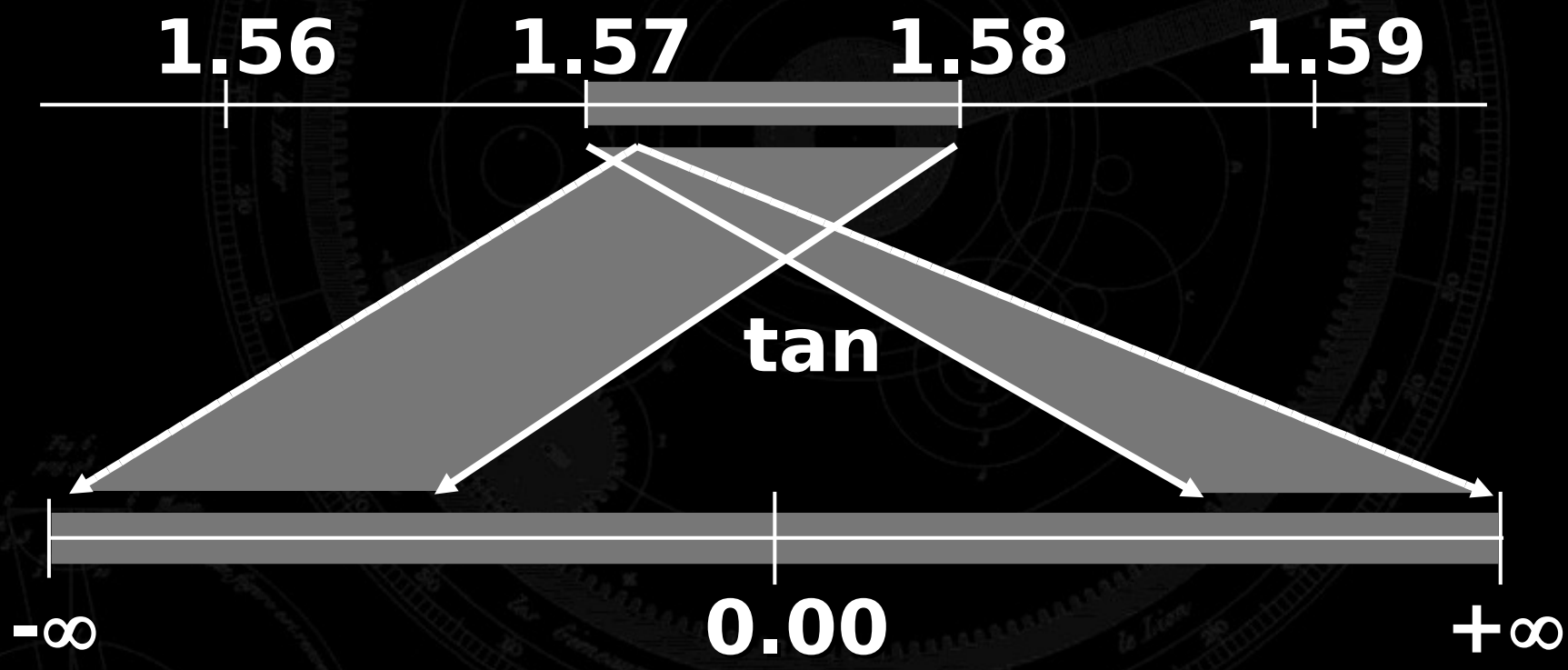


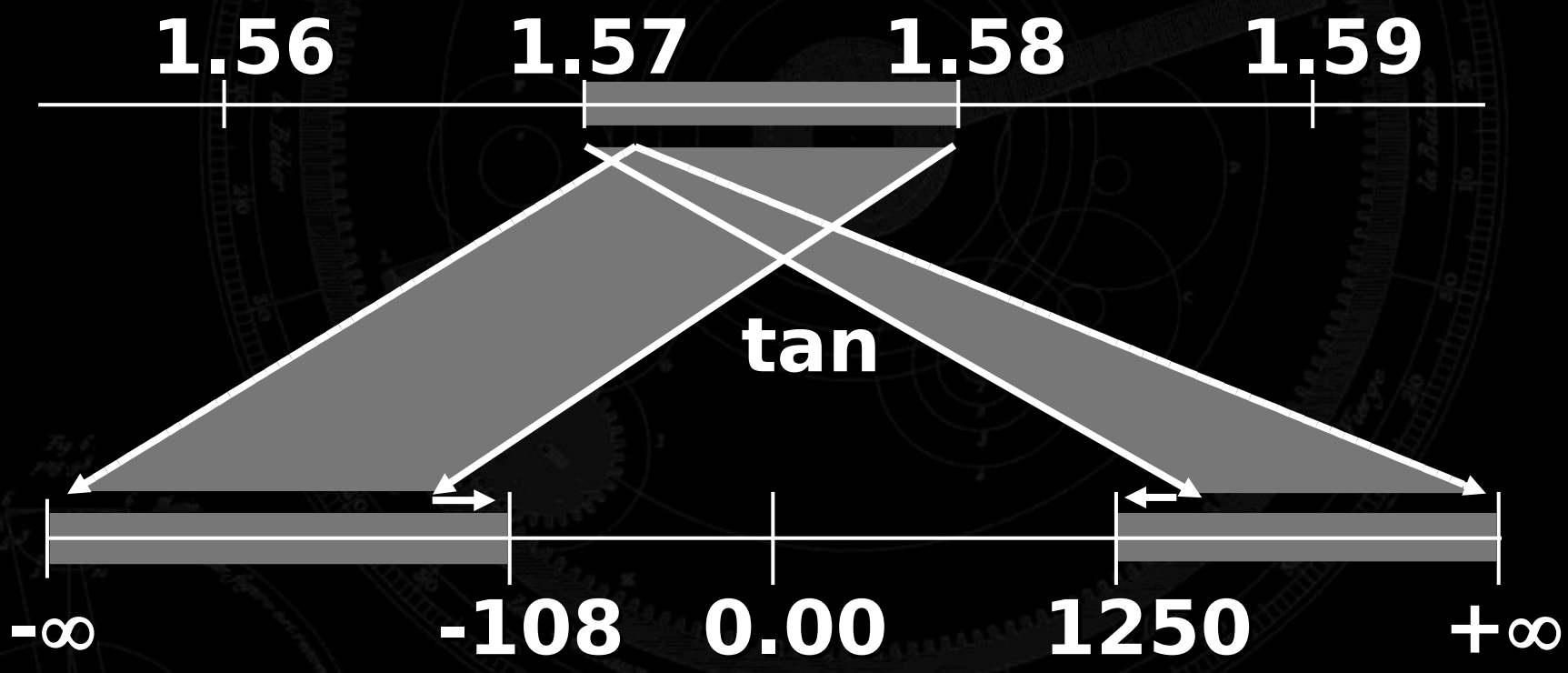
IAsolver 0.1β1 [Hickey et al.]



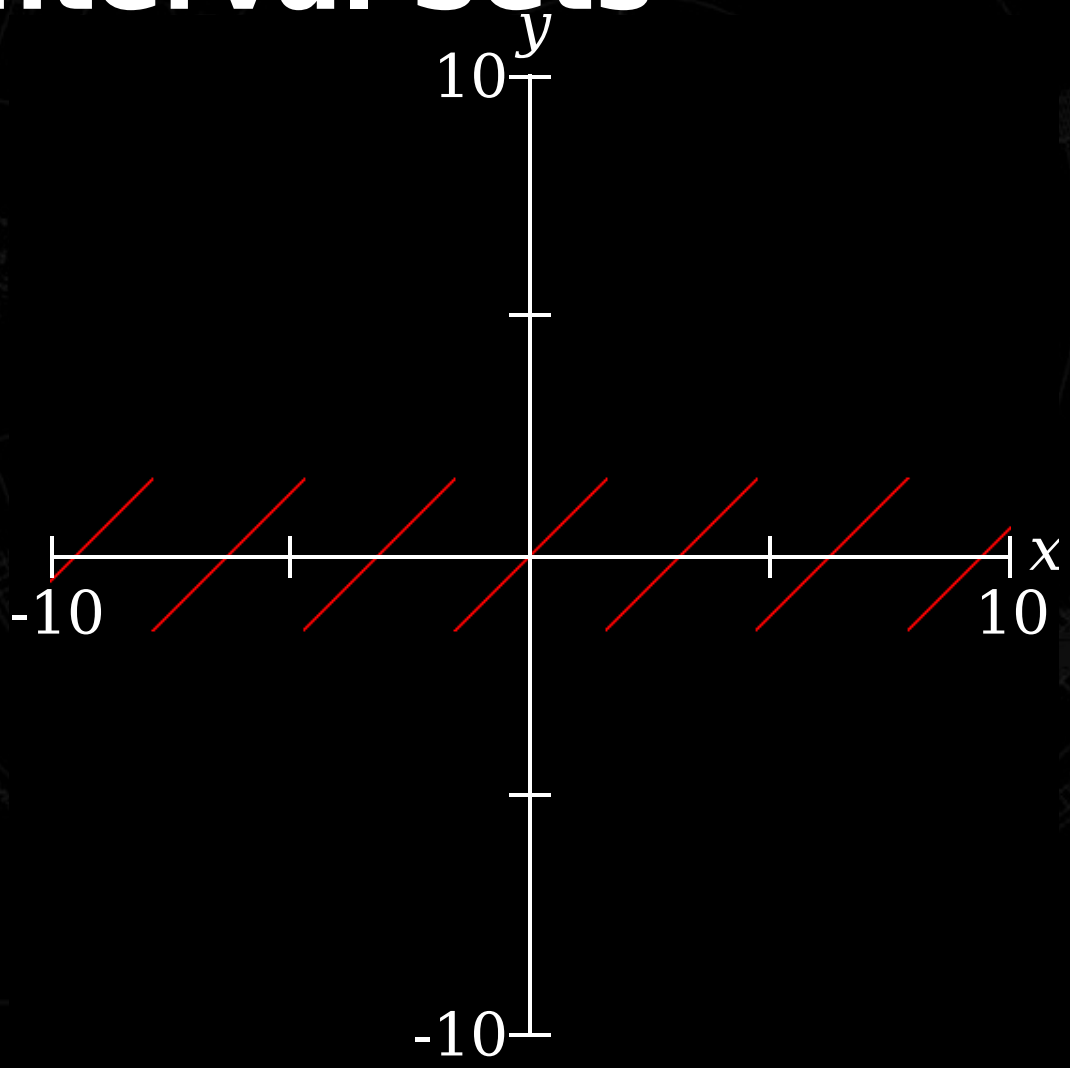
Interval Arithmetic



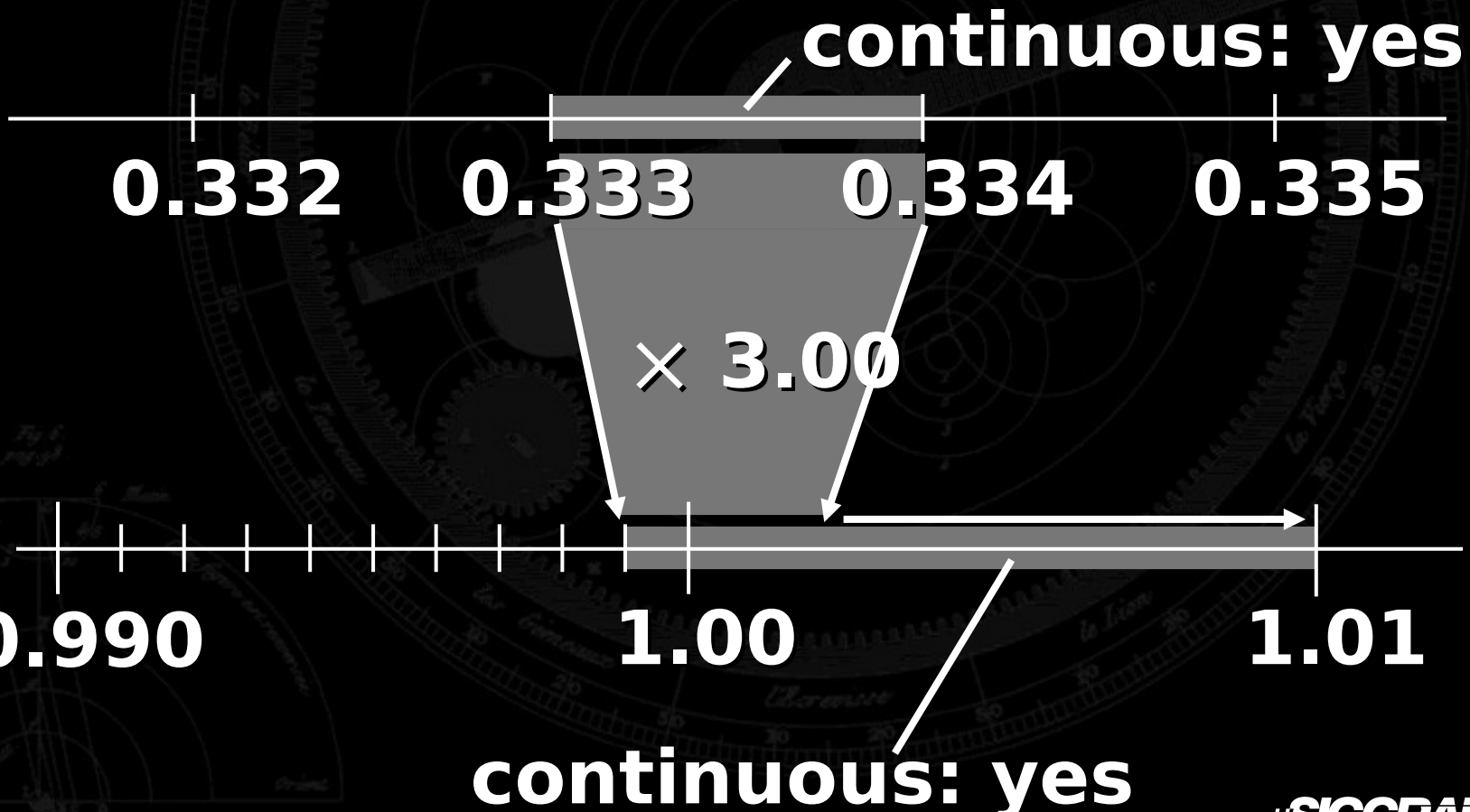
Interval Sets



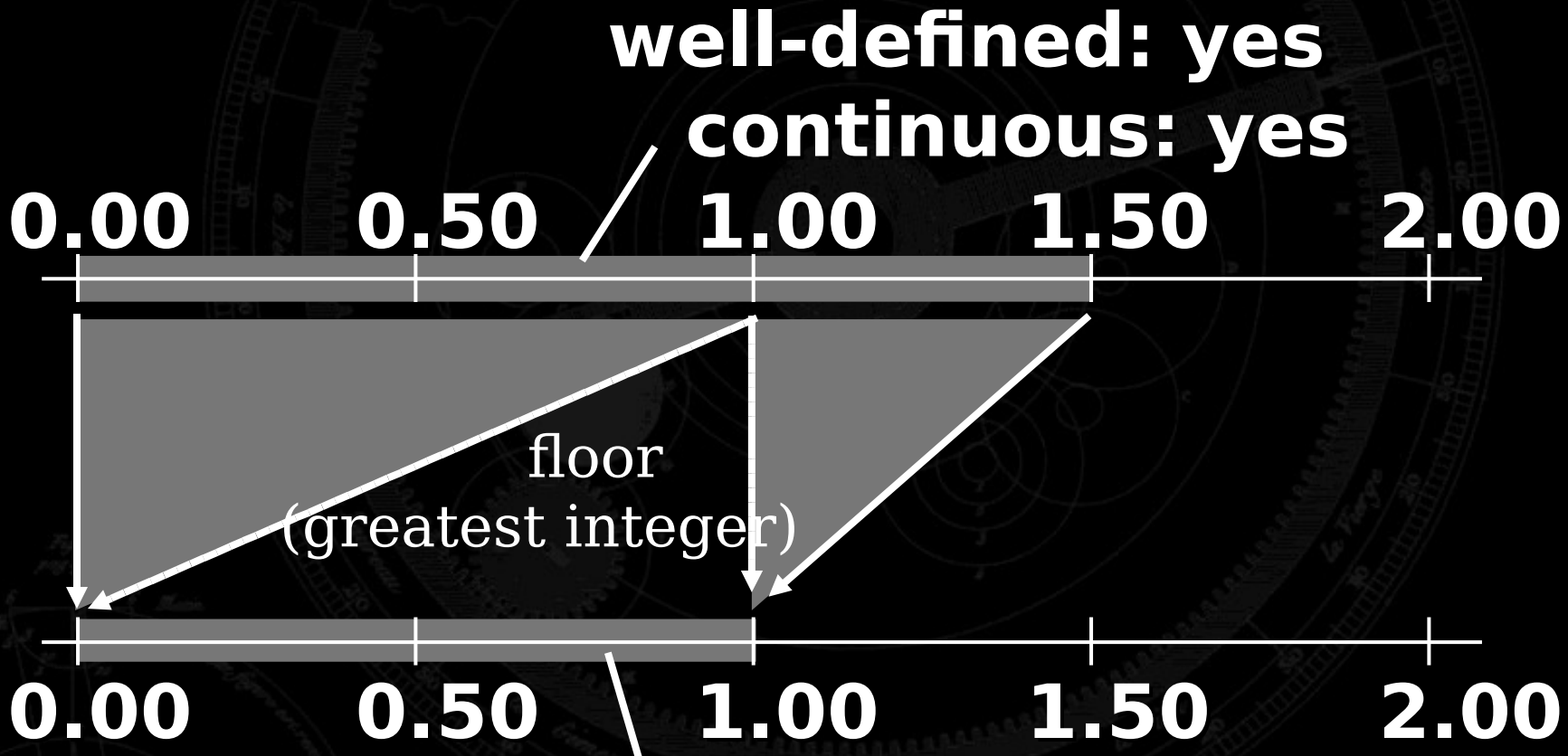
Graph From Algorithm A with Interval Sets



Continuity Tracking



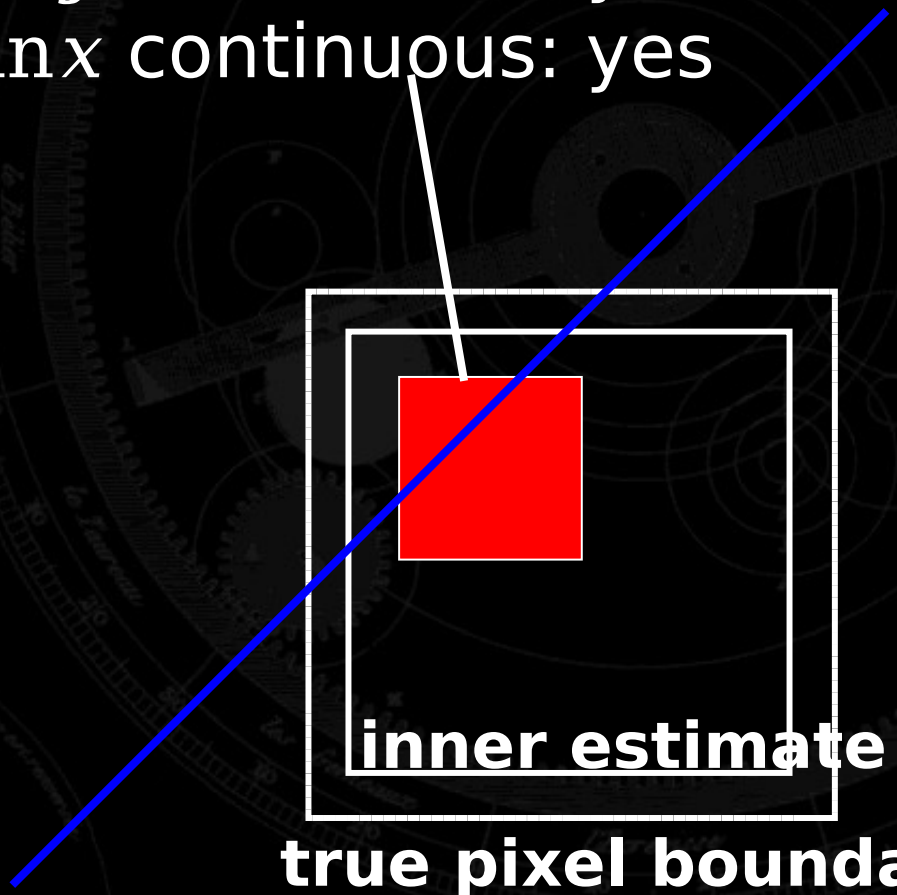
Continuity Tracking



Finding Solutions on Curves

y continuous: yes

$\mathrm{Arctan}\tan x$ continuous: yes



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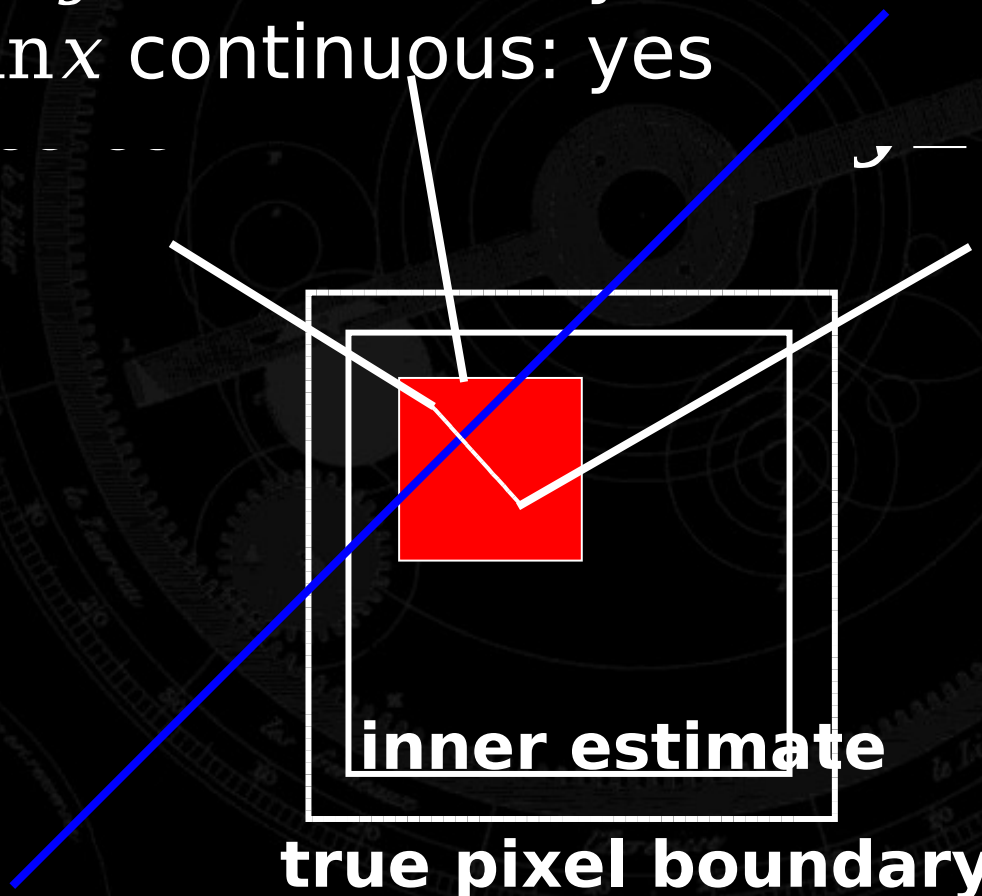
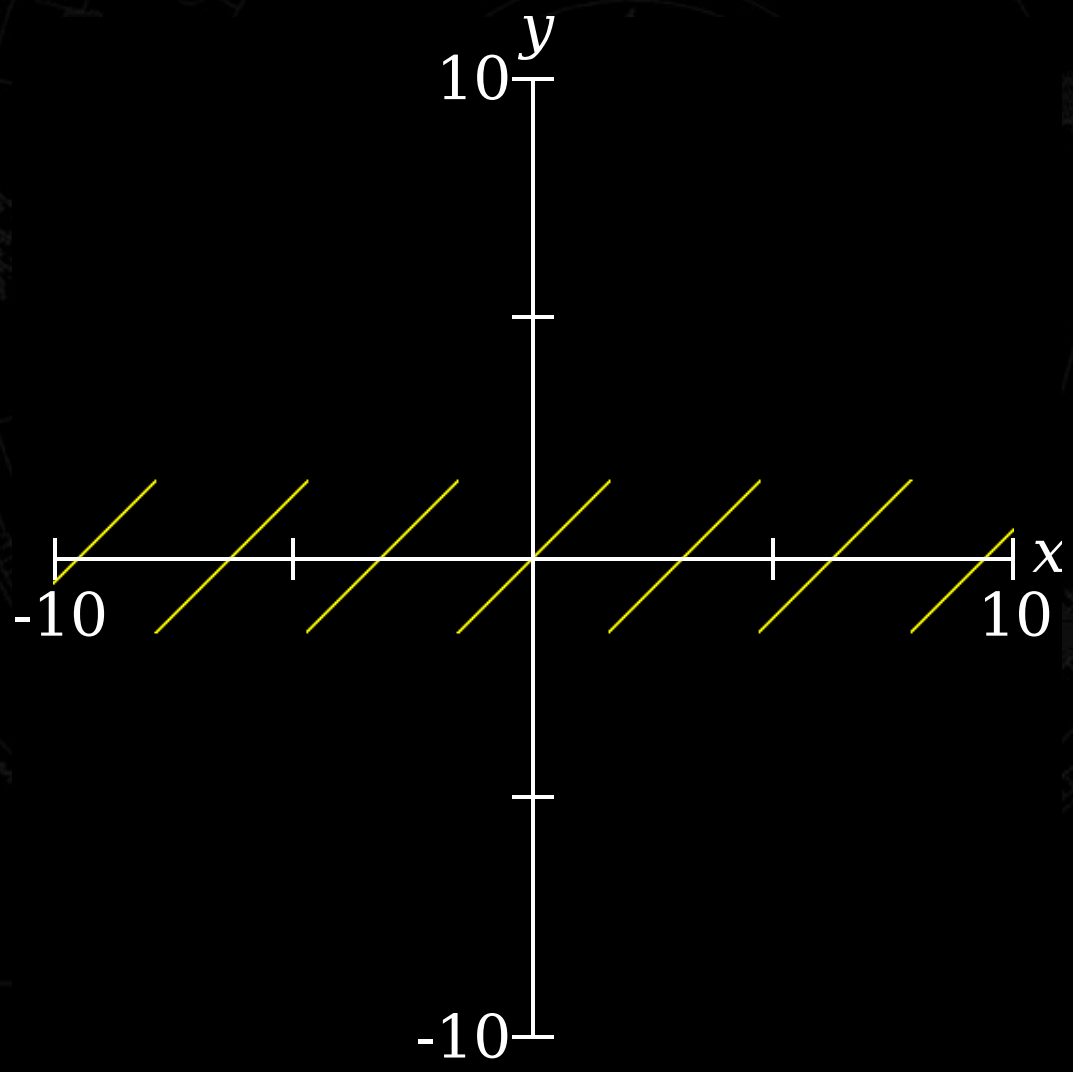


Graph from Algorithm B



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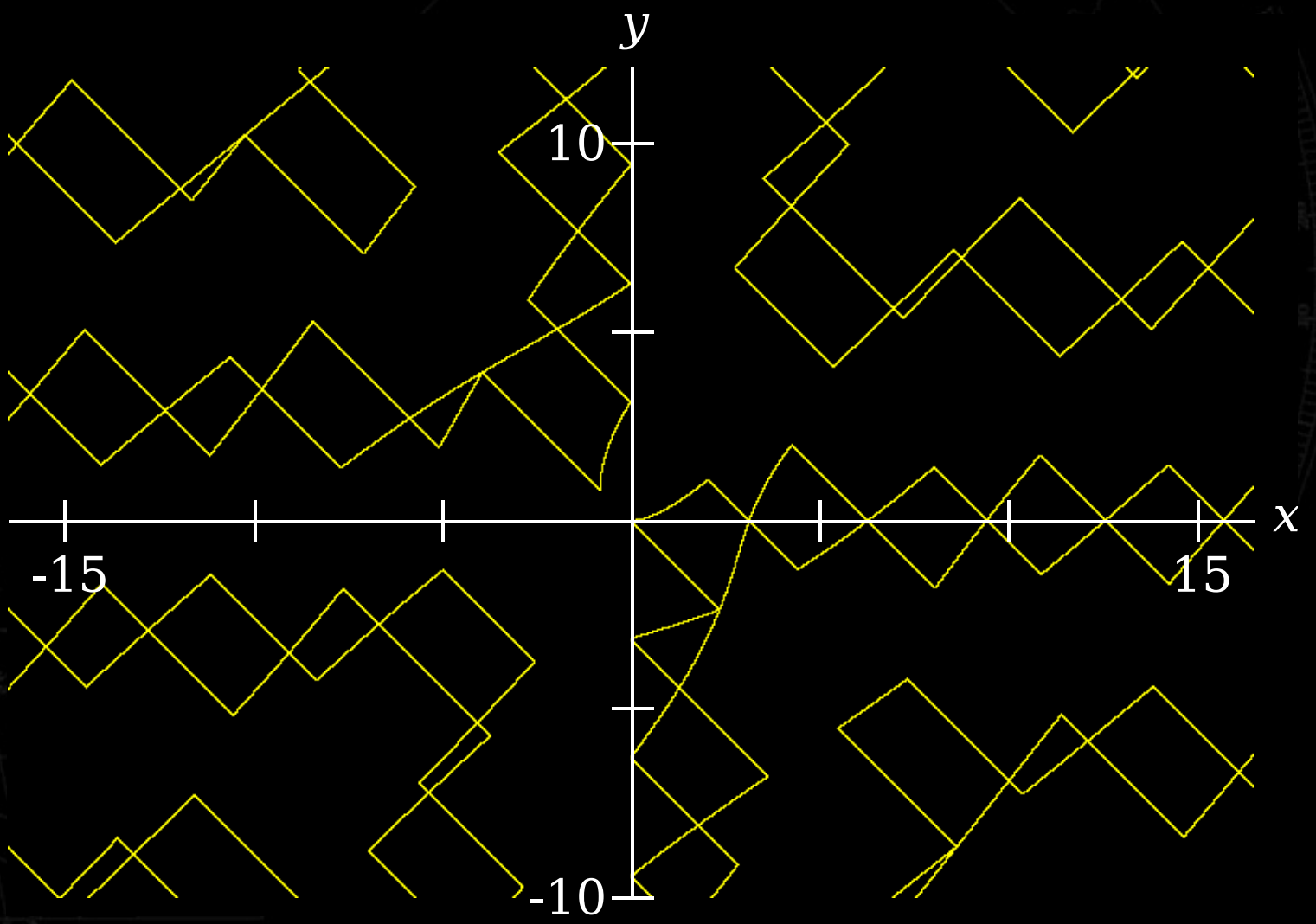

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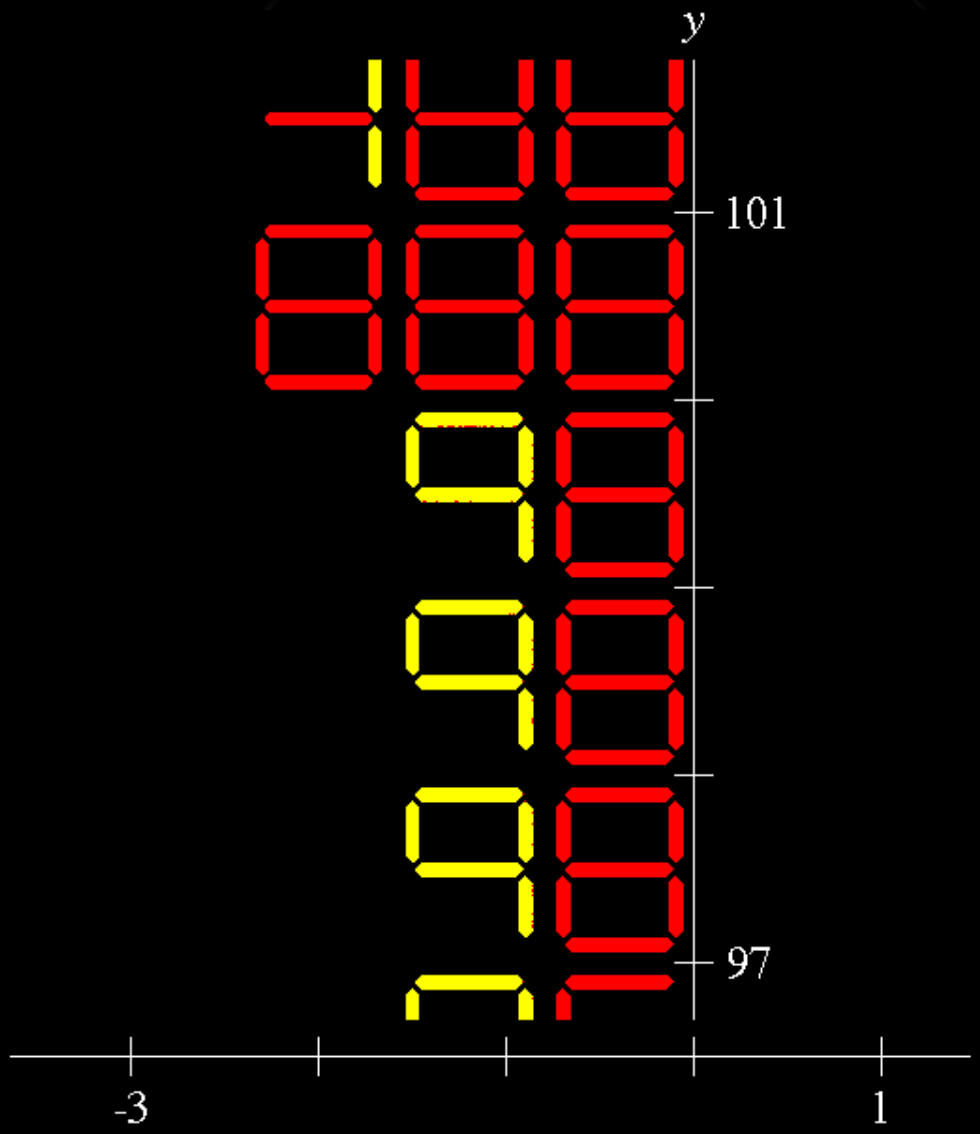
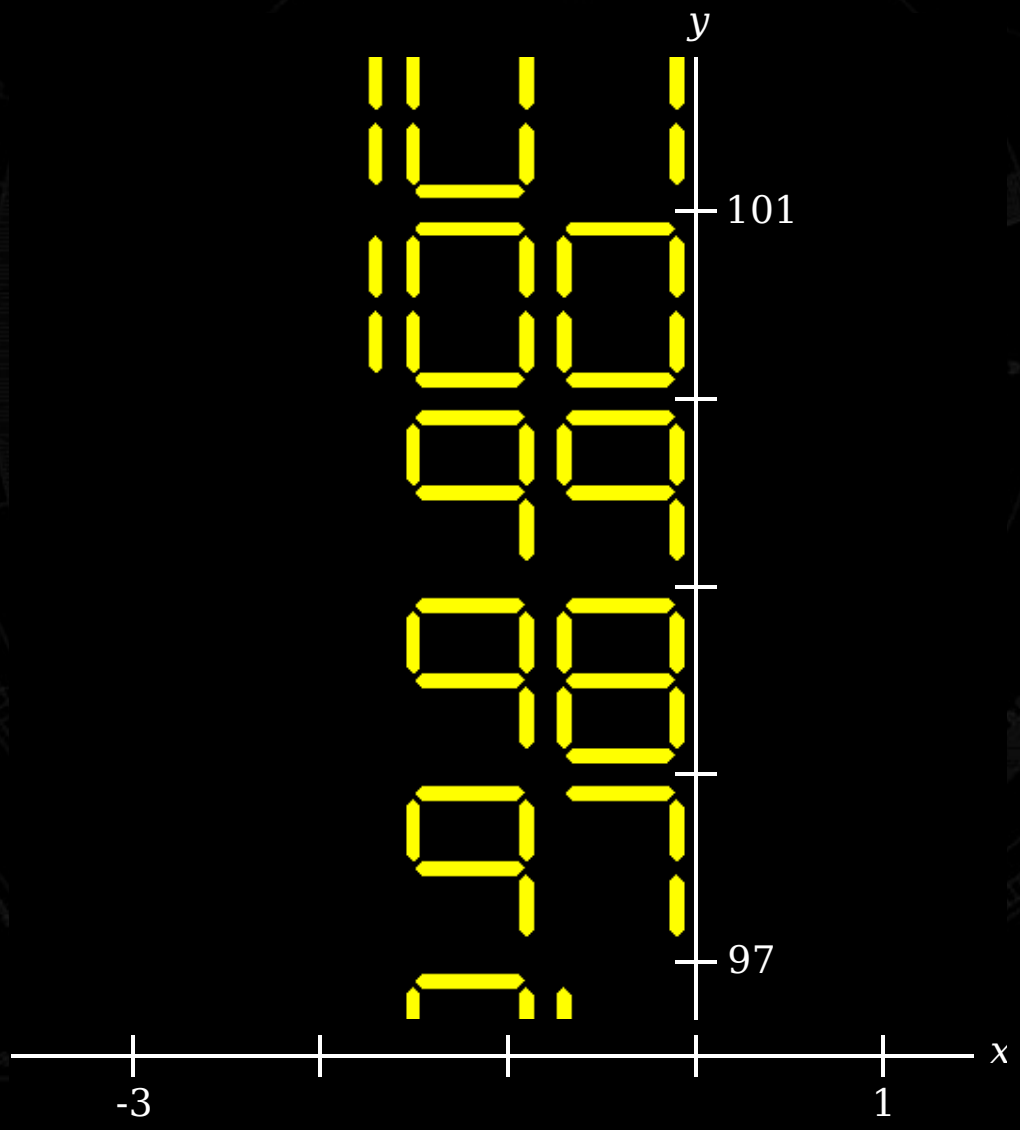


Graph from Algorithm 3.4



Conclusion

- **Most graphing programs are not reliable**
  - Reliable graphing programs do exist (GrafEq)
- **Red pixels are useful**
- **Be careful when using interval arithmetic**
  - Keeping track of the mathematical properties of evaluated formulae is possible and useful

Future Work

- **Use other colors besides red**
  - Display topological information
- **Tackle a larger class of formulae**
  - integration, differentiation, iteration, ...
- **Animation**
  - visualize role of parameters
- **3D**

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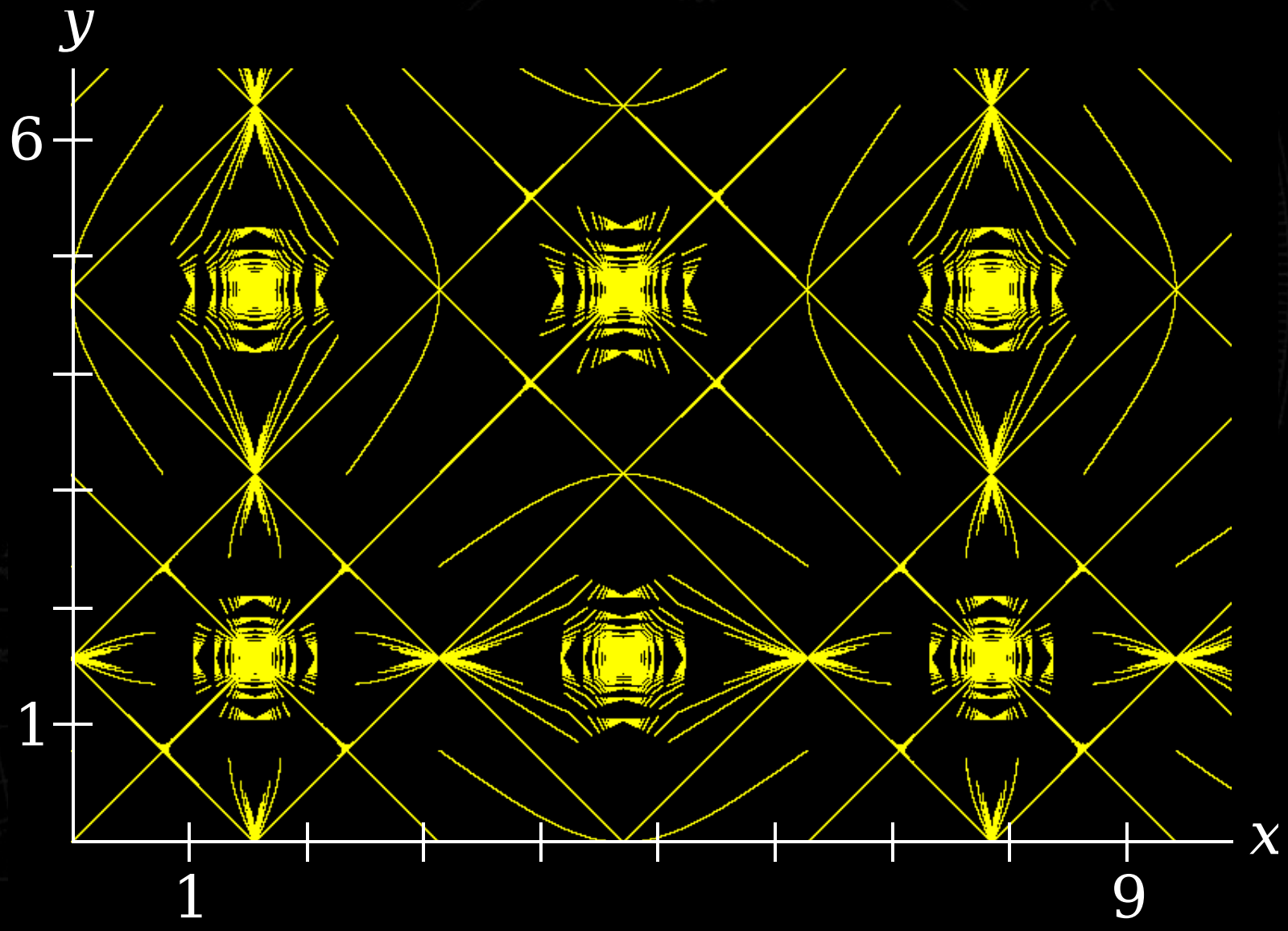
- www.peda.com/grafeq
- Creative Applications Lab 1PM-2PM Today

Example Graph



Graphing Calculator 3.0.1 [Avitzur]